Optimal Accumulation of Human and Physical Capital in A Model with Consumption Externalities and Altruism†

Been-Lon Chen‡
Academia Sinica

Yu-Shan Hsu#
National Chung Cheng University

Chih-Fang Lai*
Soochow University

June 2018

Abstract

This paper studies suboptimality of education and saving decisions in an overlapping-generation model of consumption externalities with parental altruism. Parental altruism is an important human behavior, but its role has neglected by the existing related literature. We find that parents pay education but may not leave bequests for children. Under inoperative bequest motives, equilibrium physical capital is over-accumulated, while human capital is under-accumulated. Thus, implementing public pensions and public education at the same time is called for, which is in line with the current policies employed in OECD. Optimal tax policies are envisaged in order for the equilibrium to attain the first best solution.

Keywords: Consumption externalities, Human capital, Intergenerational transfers, Bequests, Pension, Education subsidy.
JEL classification: E21; H52

---

†Earlier versions have benefitted from discussions with participants at several seminars.
‡Corresponding address: Been-Lon Chen, the Institute of Economics, Academia Sinica, 128 Academia Road Section 2, Taipei 11529, TAIWAN; Phone: (886-2)27822791 ext. 309; Fax: (886-2)27853946; email: bchen@econ.sinica.edu.tw
#Department of Economics, National Chung Cheng University, 168 University Road, Min-Hsiung, Chia-Yi 62102, TAIWAN. Phone: (886-5) 2720411 ext. 34127; Fax: (886-5) 2720816; email: ecdysh@ccu.edu.tw.
*Department of Economics, Soochow University, 56 Kuei-Yang Street Section 1, Taipei 10048, TAIWAN; Phone: (886-2)23111531 ext. 3582; Fax: (886-2)23822001; email: cflai@scu.edu.tw.

1. Introduction

This paper studies the efficiency issue of capital accumulation in an overlapping generations (OLG) economy with both physical and human capital. Since Diamond (1965), it has been well-known that physical capital accumulation path needs not be dynamically efficient. In the case of over-accumulation, intergenerational arrangements such as pensions might be useful in order to increase the welfare of all generations. Recent literature has stressed the importance of human capital on economic growth, but when human capital is taken into account, the problem becomes complicated.[1] Compared to the social optimum, in equilibrium both types of capital may under- or over-accumulate, or one type under-accumulates and the other type over-accumulates.

The issue has been studied by Boldrin and Montes (2005) and Docquier et al. (2007). In a three-period OLG model where agents borrow to invest in education,[2] Boldrin and Montes (2005) studied suboptimality of both education and saving decisions. Because of a missing credit market for human capital investment, these authors found that the long-run competitive equilibrium features under-accumulated human capital along with over-accumulated physical capital. Their paper posits a possibility wherein the levels of accumulation of physical and human capital differ from the social optimum in opposite directions. Later, Docquier et al. (2007) stress the role of intergenerational human capital externalities. These authors added these externalities into the model of Boldrin and Montes (2005) but allowed for a perfect credit market for human capital investment. They compared the competitive equilibrium with that of the social optimum, wherein the social planner's objective function is the sum of the lifetime utilities of agents over generations discounted by a social weight. For proper values of the social weight, they uncovered that the long-run equilibrium undergoes under-accumulation of human capital along with over-accumulation of physical capital, like that in Boldrin and Montes (2005). Thus, in these two pieces of work, the combination of education subsidies (like public education) and transfers from the young to the old (like public pensions) can restore efficiency and increase the welfare.

Recently, Bishnu (2013) has contended a lack of empirical backing for human capital externalities and instead advocated the importance of consumption externalities.[3] He revisited the Docquier et al. (2007) model by dropping human capital externalities and adding consumption externalities. Under this setup, he uncovered that physical and human capital either both under- or both over-accumulate relative to the

[1] For theoretical literature, see Lucas (1988) and Romer (1990), and for empirical work, Levine and Renelt (1992) and Barro and Lee (1993).

[2] Agents pay for their education when young, and work, consume and save in the middle-aged, and retire when old.

[3] Consumption externalities were emphasized in early work by Veblen (1912) and were first formalized as a determinant of aggregate consumption by Duesenberry (1949) in his development of the relative income hypothesis. Empirical studies have provided support for the significance of consumption externalities associated with social comparison; see, for example, Clark and Oswald (1996) and Luttmer (2005)

social optimum. With these results, Bishnu (2013) suggested that implementation of public pensions and public education at the same time is never recommendable. However, the recommendation is at odd with what we observe in real world. In OECD, countries always employ the two policies of public pensions and public education at the same time.[4] Such inconsistency motivates a general question: how can we justify such public intervention in education and pensions when consumption externalities are prevailing?

The purpose of this paper is to show that that parental altruism helps rationalize public involvement in pensions and education subsidies at the same time. Specifically, we study suboptimality of education and saving decisions in an otherwise Bishnu (2013) model except for altruism. In our model, parents have motives to leave bequests and finance education for their children. While leaving bequests for children is a parental transfer in goods, financing education for children is a parental transfer in kind. Both are parental altruism toward children. Parental altruism is an important human behavior, but its role has not been considered in Boldrin and Montes (2005), Docquier et al. (2007) and Bishnu (2013).

Parental bequest motives constitute an important channel for decisions on savings and capital accumulation in OLG models.[5] The literature dates back the early contribution by Barro (1974) in an OLG model, who showed that bequest motives lead to the Ricardian equivalence of the debt neutrality. Later, Buiter (1979) and Carmichael (1982) found that the debt neutrality proposition hinges on whether or not bequest motives are operative. Weil (1987) formally determined a necessary and sufficient condition for bequest motives to be operative or inoperative. Abel (1988) analyzed the effect of fiscal policy on the capital evolution under operative and inoperative bequest motives.[6]

Regarding parental finance in children's education, many existing studies find that parents are willing to pay for children's high school education and beyond (Steelman and Powell, 1991). Surveys carried out by the Sallie Mae Education Institute disclose that financing children's college education is among parents' most important investment they can make (Miller, 1997).[7] Using Consumer Expenditure Survey over the period of 1972-2007, Kornrich and Furstenberg (2007) find that about 50% of parental spending per child goes to education. Moreover, not just parents in developed countries, parents in developing

[4] For pensions in OECD, readers are referred to http://www.oecd.org/els/public-pensions, and for public spending on education, referred to https://data.oecd.org/eduresource/public-spending-on-education.htm.

[5] See Michel et al. (2006) for a survey about bequest motives.

[6] Other than pure altruism used in Barro (1974), several alternative bequest motives have been proposed, including incomplete annuity markets (Abel, 1985), strategic bequest behavior (Bernheim et al., 1985), joy of giving (Abel, 1988) and warm-glow giving (Andreoni, 1990).

[7] According to surveys conducted by Sallie Mae commissioned Gallup & Robinson, Inc. and reported by Miller (1997), most parents (92%) surveyed agree with the statement, "A college education is among the most important investment I will make for my child," and "even with what it costs today, college is still a good investment." The importance placed on their children's education is confirmed, when 31% of the parents report that children's college education is their highest financial priority, second only to the 38% of parents who indicate everyday budget is their first financial priority and almost twice the 17% of parents who name retirement as their first priority.

countries are also willing to finance children's education. For example, using 1985-1986 Peru living Standard Survey, Gertler and Glewwe (1990) reveal that parents in Peru, and even in rural areas, are willing to pay for children's education.[8]

Recently, several papers have considered altruism when analyzing various issues.[9] Closest to our paper is Alonso-Carrera et al. (2008), who analyzed suboptimal allocation of physical capital in an OLG model with consumption externalities and altruism. Our paper is different from theirs, because we also analyze the allocation of human capital arising from consumption externalities and altruism. Our paper may be thought of as extending Alonso-Carrera et al. (2008) to one with human capital investment. We envisage the role of bequest motives on whether the accumulation of physical and human capitals differs from the social optimum in *opposite* directions, an issue that cannot analyze in Alonso-Carrera et al. (2008).

We find that physical and human capital are under- or over-accumulated depends on whether or not the bequest motive is operative. When the bequest motive is operative, there is either over-accumulation or under-accumulation of both physical and human capitals, which is the same as that in Bishnu (2013), with the levels of accumulation of both types of capital different from the social optimum in *the same* direction. By contrast, when the bequest motive is inoperative, there is over-accumulation of physical capital and under-accumulation of human capital. Thus, the levels of accumulation of physical and human capitals differ from the social optimum in *opposite* directions, which is different from Bishnu (2013).

We should remark that our results of opposite accumulation directions are similar to Docquier et al. (2007), but they arise from different mechanisms. While our mechanism is an inoperative bequest motive, the mechanism in Docquier et al. (2007) is an intergenerational human capital externality. We should also comment that the suboptimality in Bishnu (2013) is based on comparisons between the Diamond (1965) model and the corresponding planner problem. His analysis does not allow one to distinguish the deviation of optimality due to consumption spillovers from the suboptimality due to bequest motives. In contrast, our suboptimality is based upon the comparison between the Barro (1974) model and the corresponding planner problem. The mechanism leading to the lack of optimality in our model depends not only on consumption externalities but also on bequest motives. When the bequest motive is operative, consumption externalities are the only source of suboptimality. However, when the bequest motive is

[8] In estimates made by Gertler and Glewwe (1990), it was found that rural Peruvian households at all income levels are willing to pay fees high enough to cover the operating costs of new schools in their villages. See similar results found by Lillard and Willis (1997) for Malaysia and by Brown (2006) for China.

[9] Page (2003) empirically tested and found the existence of an operative bequest motive. Alonso-Carrera et al. (2007) examined how consumption habits and aspirations make the willingness to leave bequests stronger or weaker. Barnett et al. (2013) investigated how bequest-giving behavior give rise to deviant generations, generations that do not leave a bequest having received an inheritance, and vice versa. Ihori et al. (2017) analyzed a model with altruism and liquidity constraints wherein children's education costs are shared between children and parents with the share determined by parents.

inoperative, the source of suboptimality comes from both consumption externalities and inoperative bequest motives. Our analysis shows how the suboptimality initiating from consumption externalities depends on inoperative bequest motives.

Finally, we investigate how the tax policy can be used to restore efficiency and increase the social welfare. We find that capital income is taxed or subsidized, depending on the externality factor being smaller or larger than unity.[10] Moreover, education expenditure is subsidized or taxed, depending on the altruism factor being smaller or larger than the social weight. Only under operative bequest motives are bequests taxed or subsidized, and they are taxed if the altruism-factor-discounted externality factor is larger than the social weight, and subsidized if otherwise. By contrast, under inoperative bequest motives, there is no bequest in equilibrium, but the optimal bequest is negative. Then, the social optimum calls for intergenerational lump-sum transfers from the working-aged to retirees if education is subsidized or taxed at an amount smaller than the absolute value of optimal bequests, and calls for intergenerational lump-sum transfers from the retirees to the working-aged if otherwise.

We organize this paper as follows. In Section 2, we set up an OLG model with altruism, with the competitive equilibrium and the social optimum analyzed in Subsections 2.1 and 2.2, respectively. In Section 3, we compare the competitive equilibrium with the social optimum. Section 4 analyzes optimal policies to restore optimality. Finally, we offer some concluding remarks in Section 5.

2. The model

Our model follows from Boldrin and Montes (2005), Docquier et al. (2007) and Bishnu (2013). The economy comprises a sequence of overlapping generations, an initial old generation, and an infinitely lived government. Agents live for three periods of life: young, middle-aged (or working-aged), and old. Specifically, our model is otherwise the same as Bishnu (2013) except for allowing for altruism wherein parents may finance young children's education and the old may leave bequests for children. In the first period of life, agents do not consume and they receive education. In the second period of life, agents work, consume, save and pay children's education. In the final period of life, agents retire, and consume and leave bequests for children.

The generation that works in period t is indexed by t. Thus, agents born in period t-1 are referred to as generation t. Let the population of generation t be N_t which grows at the rate of n, i.e., $N_t = nN_{t-1}$. Then, $N_{t-1} + N_t + N_{t+1}$ is the total population size in period t. Denote by e_{t-1} the education cost of an agent born in period t-1 (i.e., a generation t agent), which is paid by parents of generation t-1. The education

[10] While the externality factor will be defined later, it suffices to note that if the externality factor is larger than unity, then consumption externalities from the middle-aged are larger than those from the old; and vice versa.

investment results in a level of human capital in period t as described by the following process

$$h_t = \phi(e_{t-1}), \tag{1}$$

where $\phi(e)$ is a positive, strictly increasing and strictly concave function, that satisfies the Inada condition, namely, $\phi'(0)=\infty$ and $\phi'(\infty)=0$. Notice that, like Bishnu (2013) and different from Docquier et al. (2007), the human capital accumulation is free from any effect of externality.

The economy has a final good Y_t, produced by a neoclassical technology $Y_t = F(K_t, H_t)$, where K_t is aggregate physical capital and H_t is aggregate human capital, with F assumed to be homogeneous of degree 1 and satisfying the Inada condition. Denote capital per capita and human capital per capita at the beginning of period t by $k_t = \frac{K_t}{N_t}$ and $h_t = \frac{H_t}{N_t}$, respectively. Moreover, denote the ratio of physical capital per capita to human capital per capita at the beginning of period t by $\tilde{k}_t \equiv \frac{k_t}{h_t} = \frac{K_t}{H_t}$, which is the economy's average capital per unit of human capital. For simplicity, $\tilde{k}$ is referred to as effective capital per capita. Then, we can rewrite $Y_t = H_t f(\tilde{k}_t)$, where $f(\cdot) > 0$ and $f'(\cdot) > 0 > f''(\cdot)$. For simplicity, we assume that physical capital depreciates completely in one period. Assuming that the factor markets are competitive, then in optimum, firms rent capital and hire labor to levels that equalize their factor price to their marginal product.

$$R_t = R(\tilde{k}_t) \equiv f'(\tilde{k}_t),\ \ R'(\tilde{k}_t) < 0, \tag{2a}$$

$$w_t = w(\tilde{k}_t) \equiv f(\tilde{k}_t) - \tilde{k}_t f'(\tilde{k}_t),\ \ w'(\tilde{k}_t) > 0, \tag{2b}$$

where R_t is the gross return to capital in period t and w_t is the wage rate of raw labor in period t.

Let c_t and d_{t+1} denote the consumption of a generation t agent when middle-aged and old, respectively. As in Abel (2005), Alonso-Carrera et al. (2008) and Bishnu (2013), an agent obtains utility from effective consumption that compares her own consumption with a consumption reference. The felicity function is $u(\hat{c}_t, \hat{d}_{t+1})$, where $\hat{c}_t$ and $\hat{d}_{t+1}$ are an agent's effective consumption when middle-aged and old, respectively. The felicity function is strictly increasing and strictly concave, is additive in its two arguments, and satisfies the Inada condition. Effective consumption is given by $\hat{c}_t \equiv c_t - \sigma(\bar{c}_t, \bar{d}_t)$ and $\hat{d}_{t+1} \equiv d_{t+1} - \varphi(\bar{c}_{t+1}, \bar{d}_{t+1})$, where consumption references $\sigma(\bar{c}_t, \bar{d}_t)$ and $\varphi(\bar{c}_{t+1}, \bar{d}_{t+1})$ represent negative external effects caused by average consumption $\bar{c}_j$ and $\bar{d}_j$ of relevant generations j. As the existing papers, we assume that $\sigma(\bar{c}_t, \bar{d}_t)$ and $\varphi(\bar{c}_{t+1}, \bar{d}_{t+1})$ are linear with respect to $\bar{c}_t$, $\bar{d}_t$, $\bar{c}_{t+1}$ and $\bar{d}_{t+1}$, with partial derivatives σ_{c_t}, σ_{d_t}, $\varphi_{c_{t+1}}$ and $\varphi_{d_{t+1}}$ all being smaller than one. Note that σ_{c_t} and $\varphi_{d_{t+1}}$ measure the degree to which a middle-aged agent and an old agent, respectively, keeps up with the consumption of agents in the same generation. Similarly, σ_{d_t} and $\varphi_{c_{t+1}}$ gauge the degree to which a middle-aged agent and an old agent keeps up with the consumption of agents in the other generation.

Let $V(e_{t-1}, b_t)$ denote the lifetime utility of a generation t agent, who receives education at a cost e_{t-1} when young and inherits an amount of bequests b_t when middle-aged. A parent is altruistic towards children. Following Alonso-Carrera et al. (2008), the lifetime utility of a generation t agent is

$$V(e_{t-1}, b_t) = \max\left[u(\hat{c}_t, \hat{d}_{t+1}) + \beta V(e_t, b_{t+1})\right], \tag{3}$$

where $V(e_{t-1}, b_t)$ is the lifetime utility of generation t+1, and $\beta \in [0,1)$ is the altruism factor.

A working-aged agent distributes labor income $w_t h_t$ and inheritance b_t among consumption c_t, savings s_t and children's education e_t. Moreover, an old individual receives the return from savings and allocates it between consumption d_{t+1} and bequests for children b_{t+1}. Their budget constraints are, respectively,

$$w_t h_t + b_t = c_t + s_t + ne_t, \tag{4a}$$

$$R_{t+1} s_t = d_{t+1} + nb_{t+1}. \tag{4b}$$

The optimization problem of the representative generation t agent is to maximize (3) with respect to $\{c_t, s_t, e_t, b_{t+1}\}_{t=0}^{\infty}$, subject to (1), (4a) and (4b) with $b_{t+1} \geq 0$, given k_0 and h_0 and taking as given w_t and R_{t+1} for all t. The optimization conditions are as follows. First, by the envelope theorem, we obtain

$$\frac{\partial V_t}{\partial e_{t-1}} = u_{\hat{c}_t}(\hat{c}_t, \hat{d}_{t+1}) w_t \phi'(e_{t-1}), \tag{5a}$$

$$\frac{\partial V_t}{\partial b_t} = u_{\hat{c}_t}(\hat{c}_t, \hat{d}_{t+1}). \tag{5b}$$

Moreover, the optimality conditions for s_t, e_t and b_{t+1}, along with the use of (5a) and (5b), give

$$u_{\hat{c}_t}(\hat{c}_t, \hat{d}_{t+1}) = u_{\hat{d}_{t+1}}(\hat{c}_t, \hat{d}_{t+1}) R_{t+1}, \tag{6a}$$

$$nu_{\hat{c}_t}(\hat{c}_t, \hat{d}_{t+1}) \geq \beta \frac{\partial V_{t+1}}{\partial e_t} = \beta u_{\hat{c}_{t+1}}(\hat{c}_{t+1}, \hat{d}_{t+2}) w_{t+1} \phi'(e_t), \tag{6b}$$

$$nu_{\hat{d}_{t+1}}(\hat{c}_t, \hat{d}_{t+1}) \geq \beta \frac{\partial V_{t+1}}{\partial b_{t+1}} = \beta u_{\hat{c}_{t+1}}(\hat{c}_{t+1}, \hat{d}_{t+2}), \tag{6c}$$

where the equality in (6b) and (6c) holds if $e_t > 0$ and $b_{t+1} > 0$, respectively.

Condition (6a) is the Euler equation for consumption between middle and old ages. Condition (6b) is the decision for paying children's education. As $\phi(e)$ satisfies the Inada condition, (6b) holds with an equality and thus, e_t is positive. Hence, a parent invests in children's education to the level, when the loss in the marginal utility of consumption equals the gain in the (altruism-factor) discounted future marginal lifetime utility arising from children's marginal utility of consumption due to higher wage income.

Condition (6c) determines optimal bequests. If the bequest motive is operative, (6c) holds with an equality and b_t>0. Then, a parent leaves bequests to children to the level when the decrease in her marginal utility of consumption equals the increase in the (altruism-factor) discounted future marginal lifetime utility arising from children's marginal utility of consumption. By contrast, if the bequest motive is not operative, (6c) holds with an inequality and then, b_t=0.

In a symmetric equilibrium, $\bar{c}_t = c_t$, $\bar{d}_t = d_t$, $\bar{c}_{t+1} = c_{t+1}$ and $\bar{d}_{t+1} = d_{t+1}$. Moreover, with capital depreciating fully in one period, aggregate savings in period t are the capital stock at the beginning of period t+1. Thus, the goods market clearing condition is

$$s_t = nk_{t+1} = n\tilde{k}_{t+1}h_{t+1}. \tag{7}$$

2.1 Competitive equilibrium

This subsection analyzes the competitive equilibrium.

Definition 1. For given h_0 and k_0, a competitive equilibrium is the path $\{c_t, d_t, k_t, s_t, e_t, b_t, h_t, w_t, R_t\}_{t=0}^{\infty}$ that satisfies firms' optimization conditions (2a)-(2b), agents' optimization conditions (5a)-(5b) and (6a)-(6c), human capital accumulation (1), the goods market clearing condition (7), and the transversality condition $\lim_{t\to\infty} \beta^t u_{\hat{c}_t} b_t = 0.$

From now on, prices and allocations with a superscript *CE* stand for those in competitive equilibrium. Using (2a) and (2b), we rewrite the decisions of savings and education in (6a) and (6b), respectively, as *intertemporal* and *intergenerational* consumption Euler equations,

$$\frac{u_{\hat{c}_t^{CE}}}{u_{\hat{d}_{t+1}^{CE}}} = f'(\tilde{k}_{t+1}^{CE}), \tag{8a}$$

$$\frac{u_{\hat{c}_t^{CE}}}{u_{\hat{c}_{t+1}}} = \frac{\beta}{n}\phi'(e_t^{CE})w(\tilde{k}_{t+1}^{CE}). \tag{8b}$$

If condition (6c) is binding, it is rewritten as $u_{\hat{d}_{t+1}^{CE}} = \frac{\beta}{n} u_{\hat{c}_{t+1}^{CE}}$. Substituting this relation into the intertemporal consumption Euler equation (8a) gives $\frac{u_{\hat{c}_t^{CE}}}{u_{\hat{c}_{t+1}^{CE}}} = \frac{\beta}{n} f'(\tilde{k}_{t+1}^{CE})$. Then, using this equation and (2b), we can rewrite the intergenerational consumption Euler equation (8b) as a relationship between next period's effective capital $\tilde{k}_{t+1}^{CE}$ and this period's education spending e_t^{CE}, with the latter determining next period's human capital h_{t+1}^{CE}.

$$\phi'(e_t^{CE}) = \frac{f'(\tilde{k}_{t+1}^{CE})}{f(\tilde{k}_{t+1}^{CE}) - \tilde{k}_{t+1}^{CE} f'(\tilde{k}_{t+1}^{CE})}. \tag{8c}$$

In steady state, the allocations in competitive equilibrium $\{c_t^{CE}, d_t^{CE}, \tilde{k}_t^{CE}, s_t^{CE}, e_t^{CE}, b_t^{CE}, h_t^{CE}\}$ are time-invariant, denoted by $\{c^{CE}, d^{CE}, \tilde{k}^{CE}, s^{CE}, e^{CE}, b^{CE}, h^{CE}\}$. First, in the steady state, parent's education investment for children in (8b) is rewritten as

$$n = \beta w(\tilde{k}^{CE})\phi'(e^{CE}), \tag{9a}$$

which gives a positive relationship between education costs e^{CE} and effective capital $\tilde{k}^{CE}$ as follows.

$$e^{CE} = e(\tilde{k}^{CE}) \equiv (\phi')^{-1}\left(\frac{n}{\beta w(\tilde{k}^{CE})}\right), \quad e'(\tilde{k}^{CE}) = \frac{-w'(\tilde{k}^{CE})\phi'(e^{CE})}{w(\tilde{k}^{CE})\phi''(e^{CE})} > 0. \tag{9b}$$

Next, with the use of (1), (2a), (2b) and (9b), the budget constraints (4a) and (4b) enable us to write consumption at middle age and when old as functions of effective capital and bequests as follows.[11]

$$c^{CE} = c^{CE}(\tilde{k}^{CE}, b^{CE}), \tag{10a}$$

$$d^{CE} = d^{CE}(\tilde{k}^{CE}, b^{CE}), \tag{10b}$$

where $\frac{\partial c^{CE}}{\partial \tilde{k}^{CE}} < 0, \ \frac{\partial c^{CE}}{\partial b^{CE}} > 0, \ \frac{\partial d^{CE}}{\partial \tilde{k}^{CE}} > 0, \ \frac{\partial d^{CE}}{\partial b^{CE}} < 0.$ [12]

These two above relationships enable us to write effective consumption of middle-aged agents $\hat{c}^{CE}$ and old agents $\hat{d}^{CE}$ as functions of $\tilde{k}^{CE}$ and b^{CE} as follows.[13]

$$\hat{c}^{CE} = \hat{c}(\tilde{k}^{CE}, b^{CE}), \tag{11a}$$

$$\hat{d}^{CE} = \hat{d}(\tilde{k}^{CE}, b^{CE}). \tag{11b}$$

Moreover, using (2a), we rewrite (8a) as $u_{\hat{c}}(\hat{c}^{CE}, \hat{d}^{CE}) - f'(\tilde{k}^{CE})u_{\hat{d}}(\hat{c}^{CE}, \hat{d}^{CE}) = 0,$ and thus a zero net marginal cost of savings (c.f. (6a)). Using (11a) and (11b), this condition can be expressed as a function of $\tilde{k}^{CE}$ and b^{CE} as follows.[14]

$$\Gamma(\tilde{k}^{CE}, b^{CE}) \equiv u_{\hat{c}}(\hat{c}(\tilde{k}^{CE}, b^{CE})) - f'(\tilde{k}^{CE})u_{\hat{d}}(\hat{d}(\tilde{k}^{CE}, b^{CE})) = 0. \tag{11c}$$

---

[11] That is, $c^{CE}(\tilde{k}^{CE}, b^{CE}) \equiv w(\tilde{k}^{CE})\phi(e(\tilde{k}^{CE})) + b^{CE} - n\tilde{k}^{CE}\phi(e(\tilde{k}^{CE})) - ne(\tilde{k}^{CE})$ and $d^{CE}(\tilde{k}^{CE}, b^{CE}) \equiv f'(\tilde{k}^{CE})n\tilde{k}^{CE}\phi(e(\tilde{k}^{CE})) - nb^{CE}$.

[12] To derive these partial effects with respect to $\tilde{k}^{CE}$, we have followed Alonso-Carrera et al. (2008) and assumed that the negative effect of $\tilde{k}^{CE}$ on c^{CE} dominates and the positive effect of $\tilde{k}^{CE}$ on d^{CE} dominates. Thus, $\frac{\partial c^{CE}}{\partial \tilde{k}^{CE}} < 0$ and $\frac{\partial d^{CE}}{\partial \tilde{k}^{CE}} > 0$. These conditions for $\frac{\partial c^{CE}}{\partial \tilde{k}^{CE}} < 0$ and $\frac{\partial d^{CE}}{\partial \tilde{k}^{CE}} > 0$ are easily satisfied, if the utility is logarithmic, the production function is Cobb–Douglas, and the values of σ_c, σ_d, φ_c and φ_d are small.

[13] $\hat{c}(\tilde{k}^{CE}, b^{CE}) \equiv c^{CE}(\tilde{k}^{CE}, b^{CE}) - \sigma(c^{CE}(\tilde{k}^{CE}, b^{CE}), d^{CE}(\tilde{k}^{CE}, b^{CE}))$ and $\hat{d}(\tilde{k}^{CE}, b^{CE}) \equiv d^{CE}(\tilde{k}^{CE}, b^{CE}) - \varphi(c^{CE}(\tilde{k}^{CE}, b^{CE}), d^{CE}(\tilde{k}^{CE}, b^{CE}))$.

[14] Notice that $\Gamma(\tilde{k}^{CE}, b^{CE})$ is defined for $\tilde{k}^{CE}>0$ and $b^{CE}>0$ such that $\hat{c}(\tilde{k}^{CE}, b^{CE})>0$ and $\hat{d}(\tilde{k}^{CE}, b^{CE})>0$.

Differentiating $\Gamma(\tilde{k}^{CE}, b^{CE})$ yields $\Gamma_b \equiv \frac{\partial \Gamma}{\partial b^{CE}} < 0$ for all b^{CE}>0 and $\Gamma_{\tilde{k}} \equiv \frac{\partial \Gamma}{\partial \tilde{k}^{CE}} > 0$ for all b^{CE}≥0.[15] Intuitively, a higher bequest increases middle-aged agents' consumption, decreases old agents' consumption and thus, decreases the net marginal cost of savings. By contrast, a higher effective capital decreases middle-aged agents' consumption, increases old agents' consumption and thus, increases the net marginal cost of savings.

It suffices to impose the same assumption made by Abel (1987) and Alonso-Carrera et al. (2008).

Assumption A. There exists a unique $\tilde{k}^0 > 0$ satisfying $\Gamma(\tilde{k}^{CE}, 0) = 0$ with $\hat{c}(\tilde{k}^{CE}, 0) > 0$, $\hat{d}(\tilde{k}^{CE}, 0) > 0$ and $\Gamma_{\tilde{k}}(\tilde{k}^0, 0) > 0$.

This assumption posits that even with a zero bequest, there exists a positive effective capital $\tilde{k}^{CE}$ consistent with positive consumption of middle-aged and old agents in the steady state. Moreover, even with a zero bequest, $\Gamma_{\tilde{k}}(\tilde{k}^{CE}, 0)$ >0 means that $\Gamma(\tilde{k}^{CE}, 0)$ has a positive slope with respect to effective capital. It is clear that properties $\frac{\partial \hat{c}^{CE}}{\partial \tilde{k}^{CE}} < 0$ and $\frac{\partial \hat{d}^{CE}}{\partial \tilde{k}^{CE}} > 0$ ensure that Assumption A is met.

Note that Assumption A implies that $\Gamma(\tilde{k}^{CE}, 0) > 0$ if $\tilde{k}^{CE} > \tilde{k}^0$ and $\Gamma(\tilde{k}^{CE}, 0) < 0$ if $\tilde{k}^{CE} < \tilde{k}^0$. Yet, the bequest motive may or may not be operative and thus, b^{CE} may be positive or zero.

To see when the bequest motive is operative or inoperative, we substitute the binding steady-state condition for bequest choices (6c) $u_{\hat{d}^{CE}} = \frac{\beta}{n} u_{\hat{c}^{CE}}$ into the condition of a zero net marginal cost of savings (11c), and then evaluate it at a zero bequest. According to Assumption A, then there exists a threshold value of the altruism factor β consistent with a zero bequest as follows.

$$\bar{\beta} = \frac{n}{f'(\tilde{k}^0)}.$$

Thus, if agents' altruism factor β is larger than the threshold value $\bar{\beta},$ the bequest motive is operative. Intuitively, when parents care for their children at a degree higher than the threshold value, they leave bequests for children. By contrast, if β is smaller than the threshold value $\bar{\beta},$ parents care for their children at a degree less than the threshold value. Then, the bequest motive is inoperative and parents do not leave bequests.[16]

---

15 In deriving these signs, the properties in (10a) and (10b) are used.

16 The threshold value of the altruism factor is similar to the one obtained in Weil (1987). Consumption externalities was introduced into Alonso-Carrera et al. (2008), which modified the threshold value of the altruism factor through their effects on physical capital per capita. Human capital investment is introduced here, which changes the threshold value of the altruism factor through its effect on physical and human capital; that is via the

Case 1. Operative bequest motives

Now, the altruism factor β is larger than the threshold value $\bar{\beta}$, and thus $b^{CE}>0$. The intertemporal consumption Euler equation (8a) in the steady state, accompanied by (6c) with an equality, gives

$$n = \beta f'(\tilde{k}^{CE}). \tag{12a}$$

Then, in the steady state, a parent leaves bequests to the level, when the marginal cost of leaving bequests to n children equals the altruism-factor discounted marginal product of effective capital resulting from saving bequests for children. The condition implies a positive relationship between β and $\tilde{k}^{CE}$: an economy with a larger altruism factor leads to a larger effective capital per capita in the steady state.

Then, parents' education investment for children in (9a), which is also the intergenerational consumption Euler equation, accompanied by the use of (12a), yields

$$w(\tilde{k}^{CE})\phi'(e^{CE}) = f'(\tilde{k}^{CE}).$$

This condition equalizes the effective wage for children and the marginal product of effective capital resulting from saving bequests for children. This condition is also obtained in Bishnu (2013), but there are differences. While agents borrow their own education costs in Bishnu (2013), here parents pay for children's education costs and have operative bequest motives. As a result, the marginal product of effective capital is due to the gross loan interest payment in Bishnu (2013), but it is due to the decrease in a parent's savings resulting from saving bequests for children in our model. Note that, with the use of (2b), the above condition is rewritten as

$$[f(\tilde{k}^{CE}) - \tilde{k}^{CE} f'(\tilde{k}^{CE})]\phi'(e^{CE}) = f'(\tilde{k}^{CE}), \tag{12b}$$

and it is easy to show that $\frac{de^{CE}}{d\tilde{k}^{CE}} > 0$.

Thus, when bequest motives are operative, (12a) and (12b) characterize steady-state effective capital and education cost, and thus human capital, in competitive equilibrium.

Case 2. Inoperative bequest motives

In this case, the altruism factor β is smaller than the threshold value. Then, the intertemporal consumption Euler equation (8a) in the steady state, accompanied by (6c) with an inequality, gives

$$n > \beta f'(\tilde{k}^{CE}), \tag{13a}$$

in which a parent's marginal cost from leaving bequests to n children is larger than the altruism-factor discounted marginal product of effective capital resulting from saving bequests for children. Thus, a

effect on effective capital per capita $\tilde{k}^0$.

parent leaves no bequests to children, b^{CE}=0.

Then, parents' education investment for children in (9a) yields

$$w(\tilde{k}_t^{CE})\phi'(e^{CE}) > f'(\tilde{k}^{CE}).$$

Now, with an inoperative bequest motive, the effective wage for children is larger than the marginal product of effective capital resulting from saving bequests for children. Thus, parents pay for children's education but leave no bequests for children. Note that Bishnu (2013) does not get this condition.

With the use of (2b), the above condition is rewritten as

$$[f(\tilde{k}^{CE}) - \tilde{k}^{CE} f'(\tilde{k}^{CE})]\phi'(e^{CE}) > f'(\tilde{k}^{CE}). \tag{13b}$$

2.2 Social planner's allocations

This subsection studies the problem of a social planner (SP). A social planner takes into account the consumption externalities overlooked by individuals. A planner's resource constraint in period t is

$$h_t f(\tilde{k}_t) = c_t + \frac{d_t}{n} + n\left(e_t + k_{t+1}\right). \tag{14}$$

A social planner maximizes the sum of lifetime utilities over generations subject to human capital accumulation in (1) and the resource constraint in (14) for all t. As in Docquier et al. (2007) and Bishnu (2013), the sum of lifetime utilities over generations is discounted by a factor $\lambda \in (0, 1)$. Like these authors, this discount factor is referred to as the social weight that a social planner attaches to future generations. A high λ indicates a smaller social discount rate in that the social planner devalues less of future generations. In the analysis that follows, a social planner with a different social weight λ may be interpreted as a different social planner with each being indexed by her own λ. The Lagrangian of a social planner's problem is as follow:

$$L \equiv \sum_{t=0}^{\infty} \lambda^t \left\{ u(\hat{c}_t, \hat{d}_{t+1}) + q_t \left[\phi(e_{t-1}) f(\tilde{k}_t) - c_t - \frac{d_t}{n} - n\left(e_t + k_{t+1}\right) \right] \right\},$$

where $\lambda^t q_t$ is the multiplier associated with the economy's resource constraint in period t.

The first-order conditions with respect to $\{c_t, d_t, e_t, k_{t+1}\}_{t=0}^{\infty}$ are, respectively,

$$\lambda u_{\hat{c}_t^{SP}}\left(1-\sigma_{c_t}\right) - u_{\hat{d}_t^{SP}} \varphi_{c_t^{SP}} - \lambda q_t = 0, \tag{15a}$$

$$-\lambda u_{\hat{c}_t^{SP}} \sigma_{d_t} + u_{\hat{d}_t^{SP}}\left(1-\varphi_{d_t^{SP}}\right) - \lambda \frac{q_t}{n} = 0, \tag{15b}$$

$$\lambda q_{t+1}\phi'(e_t^{SP})\left(f(\tilde{k}_{t+1}^{SP}) - f'(\tilde{k}_{t+1}^{SP})\tilde{k}_{t+1}^{SP} \right) - nq_t = 0, \tag{15c}$$

$$\lambda q_{t+1} f'(\tilde{k}_{t+1}^{SP}) - nq_t = 0, \tag{15d}$$

where superscript SP is used to stand for the planner's outcome.

Definition 2. For a given social weight λ, and initial stock h_0 and k_0, the social optimum is the path $\left\{c_t^{SP}, d_t^{SP}, k_t^{SP}, e_t^{SP}, h_t^{SP}\right\}_{t=0}^{\infty}$ that satisfies the resource constraint (14), the first-order conditions (15a)-(15d), and the transversality condition $\lim_{t\to\infty} \lambda^t q_t k_t^{SP} = 0.$

From (15a) and (15b), we obtain the social planner's *intratemporal* consumption Euler equation.

$$\frac{u_{\hat{c}_t^{SP}}}{u_{\hat{d}_t^{SP}}} = \frac{1}{\lambda}\Delta_t, \tag{16}$$

where $\Delta_t \equiv \frac{n(1-\varphi_{d_t})+\varphi_{c_t}}{(1-\sigma_{c_t})+n\sigma_{d_t}}$ is the "externality factor," which is time invariant, the reason being that σ and φ both are linear, so σ_{ct}, σ_{dt}, φ_{ct} and φ_{dt} are time invariant. Hence, hereafter, σ_{ct}, σ_{dt}, φ_{ct}, φ_{dt} and Δ_t will be denoted by σ_c, σ_d, φ_c, φ_d and Δ, respectively.

In (16), the social planner internalizes the consumption externality. If there is no externality, $\Delta = n$ and the social weight λ affects whether a social planner allocates more or less resources to future generations. With consumption externalities, both the social weight and consumption externalities influence whether a social planner would allocate more or less resources to future generations.

With the intratemporal consumption Euler equation (16), we can use (15a) and (15d) to derive the social planner's *intertemporal* and *intergenerational* consumption Euler equations, respectively, as follows.

$$\frac{u_{\hat{c}_t^{SP}}}{u_{\hat{d}_{t+1}^{SP}}} = \frac{\Delta}{n} f'(\tilde{k}_{t+1}^{SP}), \tag{17a}$$

$$\frac{u_{\hat{c}_t^{SP}}}{u_{\hat{c}_{t+1}^{SP}}} = \frac{\lambda}{n} f'(\tilde{k}_{t+1}^{SP}). \tag{17b}$$

Moreover, we use (15c) and (15d) to obtain a different intergenerational consumption Euler equation that relates next period's effective capital $\tilde{k}_{t+1}^{SP}$ to this period's education spending e_t^{SP}, which determines next period's human capital h_{t+1}^{SP}.

$$\phi'(e_t^{SP}) = \frac{f'(\tilde{k}_{t+1}^{SP})}{f(\tilde{k}_{t+1}^{SP}) - f'(\tilde{k}_{t+1}^{SP})\tilde{k}_{t+1}^{SP}}. \tag{17c}$$

In the steady state, (16) and (17a) are equal, which lead to $\frac{u_{\hat{c}^{SP}}}{u_{\hat{d}^{SP}}} = \frac{1}{\lambda}\Delta = \frac{\Delta}{n} f'(\tilde{k}^{SP}),$ and thus

$$n = \lambda f'(\tilde{k}^{SP}), \tag{18a}$$

which is the modified golden rule condition. This condition determines the golden rule effective capital per capita $\tilde{k}^{SP}$. Note that if a social planner assigns a larger social weight to future generations (a larger λ), there would be a larger optimal effective capital per capita in the long run.

Finally, in a steady state, (17c) is

$$[f(\tilde{k}^{SP}) - f'(\tilde{k}^{SP})\tilde{k}^{SP}]\phi'(e^{SP}) = f'(\tilde{k}^{SP}), \tag{18b}$$

and it is easy to show that $\frac{de^{SP}}{d\tilde{k}^{SP}} > 0$.

Thus, (18a) and (18b) characterize steady-state effective capital and education cost in the social optimum.

3. Competitive Equilibrium vs Social Optimum

This section studies whether or not physical and human capital in competitive equilibrium are equal to what a social planner would have intended. The issue dates back Diamond (1965), who showed that in the case of over-accumulation of physical capital, intergenerational rearrangement such as pensions is valuable in increasing the welfare of all generations. However, the present context is more complicated, as it encompasses not only human capital as a choice variable but also consumption externalities within and across generations. From the previous section, it is clear that the planner's choice of social weights plays a central role in determining the optimal level of physical and human capital accumulation. There, the planner solves a dynamic-planning problem with declining social weights over future generations. As such, if we are to compare the allocations between the competitive equilibrium and the social optimum, we need to find a way to make the laissez-faire allocation and the social optimum directly comparable.

The intertemporal and the intergenerational consumption Euler equations in the competitive equilibrium are (8a) and (8c) and those in the social optimum are (17a) and (17c). While (8c) is the same as (17c), (8a) is different from (17a). In comparing these two equations, we observe that their differences lie in the effect of consumption externalities overlooked by individuals as summarized in the externality factor Δ. These two optimality conditions coincide, only when there is no consumption externality and thus, $\Delta = n$. However, in the presence of consumption externalities, $\Delta \neq n$ and thus, it does not guarantee that $\tilde{k}^{CE}$ and $\tilde{k}^{SP}$ are identical. As such, although (8c) and (17c) look identical, the allocation of human capital in the laissez-faire equilibrium is different from the social optimum, if $\tilde{k}^{CE}$ and $\tilde{k}^{SP}$ are different.

With consumption externalities and to make meaningful comparisons, we follow Bishnu (2013) and establish a common point by devising the notion of a "laissez-faire supported" social weight, denoted by $\bar{\lambda}$. By construction, at this specific social weight, if there is no externality, the planner's allocation coincides with the laissez-faire allocation. However, in the presence of consumption externalities, at this specific social weight, the laissez-faire allocation may differ from the planner's allocation.

We begin our comparisons by contrasting competitive allocations with those preferred by utilitarian planners at the laissez-faire supported social weight. Then, we proceed to comparisons with the social weight different from this laissez-faire supported social weight. We focus on the comparisons in the steady state. We start with the case when the bequest motive is operative.

3.1 Suboptimality of the equilibrium when the bequest motive is operative

In the steady state, competitive equilibrium conditions (8a) and (8c) become (12a) and (12b), and the social optimum conditions (17a) and (17c) become (18a) and (18b). Note that, in comparing (12a) and (18a), $\tilde{k}^{CE}$ is equal to or different from $\tilde{k}^{SP}$ when λ is equal to or different from β. Next, e^{CE} in condition (12b) and e^{SP} in (18b) are the same or different, if $\tilde{k}^{CE}$ is equal to or different from $\tilde{k}^{SP}$.

Denote by $\eta^{i}_{j,e}$, j= k, h, i= CE, SP, the percentage change in physical capital and human capital when the education expenditure is increased by one percent. That is, $\eta^{i}_{k,e} \equiv \frac{\partial k^i}{k^i} / \frac{\partial e^i}{e^i}$ and $\eta^{i}_{h,e} \equiv \frac{\partial h^i}{h^i} / \frac{\partial e^i}{e^i}$. We can establish the following result.

Lemma 1. *For i= CE, SP, $\eta^{i}_{k,e} > \eta^{i}_{h,e}$ holds. For any λ, $\frac{d\tilde{k}^{SP}}{d\lambda} > 0$ and $\frac{de^{SP}}{d\lambda} > 0$. Moreover, if there exists a laissez-faire supported social weight $\bar{\lambda}$, it must be unique.*

To show the first part, if we differentiate (12b) with respect to k^{CE} and e^{CE} for a given h^{CE}, and also differentiate (12b) with respect to h^{CE} and e^{CE} for a given k^{CE}, then we obtain $\eta^{CE}_{k,e} > \eta^{CE}_{h,e}$.[17] Moreover, we differentiate (18b) to get $\eta^{SP}_{k,e} > \eta^{SP}_{h,e}$. Hence, $\eta^{i}_{k,e} > \eta^{i}_{h,e}$, i= CE, SP. For the second part, we differentiate (18a) to obtain $\frac{d\tilde{k}^{SP}}{d\lambda} > 0$. Moreover, differentiating (18b) yields $\frac{de^{SP}}{d\tilde{k}^{SP}} > 0$, implying $\frac{de^{SP}}{d\lambda} > 0$.

Finally, to prove the third part, we rewrite $\tilde{k}^{SP}$ in (18a) as a function of λ: $\tilde{k}^{SP} = \tilde{k}^{SP}(\lambda) \equiv (f')^{-1}(\frac{n}{\lambda})$. As $\lambda \in (0, 1)$, then $\lim_{\lambda \to 0} \tilde{k}^{SP}(\lambda) = 0$, $\lim_{\lambda \to 1} \tilde{k}^{SP}(\lambda) = (f')^{-1}(n) \equiv \tilde{k}^{SP}_{\max}$ and thus, $\tilde{k}^{SP}(\lambda) \in (0,\ \tilde{k}^{SP}_{\max})$. Suppose that there exists a laissez-faire supported social weight $\bar{\lambda}$. Since $\tilde{k}^{SP}$ strictly increases in λ, it follows that there exists a unique level of $\tilde{k}^{SP}$ that corresponds to $\bar{\lambda}$. If we denote by $\tilde{k}^{SP}(\bar{\lambda})$ the level of $\tilde{k}^{SP}$ corresponding to $\bar{\lambda}$, then $\tilde{k}^{SP}(\bar{\lambda}) \in (0,\ \tilde{k}^{SP}_{\max})$.

In words, the first part says that if the education expenditure increases by one percent, the

[17] Differentiating (12b) yields $\frac{d\ln k^{CE}}{d\ln e^{CE}} - \frac{d\ln h^{CE}}{d\ln e^{CE}} \equiv \eta^{CE}_{k,e} - \eta^{CE}_{h,e} = \frac{e^{CE}(f - \tilde{k}^{CE} f')\phi''}{\tilde{k}^{CE}(f'' - w'\phi')} > 0$, as $f - \tilde{k}^{CE} f' = w > 0$, $\phi'' < 0$, and $f'' - w'\phi' < 0$. As a result, $\eta^{CE}_{k,e} > \eta^{CE}_{h,e}$.

proportional increase of physical capital per capita is larger than the proportional increase of human capital per capita. The second part says that both effective capital per capita and education investment in the social optimum increase in the social weight. Finally, the last part says that, given a social optimum and thus the level $\tilde{k}^{SP}$, there is a unique $\bar{\lambda}$ for each possible Δ, where $\bar{\lambda}$ depends on Δ.

Other than the uniqueness result of $\tilde{k}^{SP}(\bar{\lambda})$, we also establish a uniqueness result.

Lemma 2. *For any Δ and under operative bequest motives, if there exists a* $\lambda = \lambda^*$ *such that* $\tilde{k}^{SP}(\lambda^*) = \tilde{k}^{CE}$ *holds, then* $e^{SP}(\lambda^*) = e^{CE}$, *and vice versa.*

The uniqueness result in Lemma 2 follows directly from (12b) and (18b), because there is no spillover from human capital and the form of (12b) for *CE* is the same as that of (18b) for *SP*.

We are ready to compare the allocations in *CE* with those in *SP*. As in Bishnu (2013), we will focus on $\tilde{k}$, the ratio of physical capital k to human capital h. Based on the results of Lemma 2, one can easily verify that given feasibility, h and k in *CE* and *SP* coincide at a unique social weight. According to the uniqueness results in Lemma 2, we start our analysis with the special laissez-faire supported social weight $\bar{\lambda}$. This specifies the weight at which the allocations in *CE* and *SP* are identical in an economy without consumption externalities. However, with consumption externalities, although the marginal rate of substitution in *CE* is still the same as in *SP* at $\lambda = \bar{\lambda}$, the accumulated levels of h and k in *CE* are different from the *SP* at this specific social weight. Thus, the above result no longer remains valid.

In the rest of the analysis, we proceed by varying the value of λ until we arrive at that specific social weight, where the levels of h and k in *SP* are identical to those in *CE*. For simplicity, we follow Bishnu (2013) and assume no population growth from now on; thus n=1. Then, Δ> (*resp.* = *or* <) n=1 if $(\sigma_c + \varphi_c) >$ (*resp.* = *or* <) $(\sigma_d + \varphi_d)$; that is, if the consumption externality from the middle-aged dominates (*resp.* equals *or* is dominated by) that from the old, then Δ> (*resp.* = *or* <) 1.

Case 1. **The social weight is equal to the laissez-faire supported social weight**

Starting with the case of $\lambda = \bar{\lambda}$, if we compare (12a) in *CE* with (17a) in *SP*, it is clear to see that (12a) is different from (17a) when Δ≠1. First, when Δ>1, it is obvious that $\tilde{k}^{SP}(\bar{\lambda}) > \tilde{k}^{CE}$, which implies $e^{SP}(\bar{\lambda}) > e^{CE}$ and thus, $h^{SP}(\bar{\lambda}) > h^{CE}$, according to Lemma 1. Moreover, the results of $\tilde{k}^{SP}(\bar{\lambda}) > \tilde{k}^{CE}$ and $h^{SP}(\bar{\lambda}) > h^{CE}$ imply $k^{SP}(\bar{\lambda}) > k^{CE}$. Next, when Δ<1, it is easy to see that $\tilde{k}^{SP}(\bar{\lambda}) < \tilde{k}^{CE}$, $h^{SP}(\bar{\lambda}) < h^{CE}$ and $k^{SP}(\bar{\lambda}) < k^{CE}$. Thus, the allocation in *CE* differs from that in *SP* in the same direction, with under-accumulating human and physical capital in the *CE* when Δ>1 and over-accumulating human and physical capital in *CE* when Δ<1.

Case 2. The social weight is different from the laissez-faire supported social weight

Now, we proceed to the case $\lambda \neq \bar{\lambda}$. First, when Δ<1, according to Lemma 1, wherein $\frac{d\tilde{k}^{SP}}{d\lambda} > 0$ and $\frac{de^{SP}}{d\lambda} > 0$ and thus, $\frac{dh^{SP}}{d\lambda} > 0$, there exist $\lambda_+^1 > \bar{\lambda}$ and $\lambda_+^2 > \bar{\lambda}$ such that $\tilde{k}^{SP}(\lambda_+^1) = \tilde{k}^{CE}$ and $h^{SP}(\lambda_+^2) = h^{CE}$. Besides, according to Lemma 2, we can get $\lambda_+^1 = \lambda_+^2 = \lambda_+ > \bar{\lambda}$, such that $k^{SP}(\lambda_+) = k^{CE}$ holds. It follows that, for all $\lambda > \lambda_+$, we have $h^{SP} > h^{CE}$ and $k^{SP} > k^{CE}$; and for all $\lambda < \lambda_+$, we have $h^{SP} < h^{CE}$ and $k^{SP} < k^{CE}$. Next, when Δ>1, it is easy to show that there exists $\lambda_- < \bar{\lambda}$ such that $h^{SP}(\lambda_-) = h^{CE}$, $\tilde{k}^{SP}(\lambda_-) = \tilde{k}^{CE}$ and $k^{SP}(\lambda_-) = k^{CE}$ hold. It follows that, for all $\lambda > \lambda_-$, we have $h^{SP} > h^{CE}$ and $k^{SP} > k^{CE}$; and for all $\lambda < \lambda_-$, we have $h^{SP} < h^{CE}$ and $k^{SP} < k^{CE}$.

Thus, when the social weight λ differs from the laissez-faire supported social weight $\bar{\lambda}$, the allocation in *CE* also differs from the *SP* in the same direction, with over-accumulating human and physical capital in *CE* when Δ<1, and under-accumulating human and physical capital in *CE* when Δ>1.

To summarize these results, we have the following proposition.

Proposition 1. *At any λ and the presence of consumption externalities, when the bequest motive is operative, the levels of physical and human capital in competitive equilibrium are different from those in the social optimum in the same directions.*

Thus, when the bequest motive is operative, no matter whether the social weight is equal to the laissez-faire supported social weight or not $(i.e., \lambda = \bar{\lambda} \text{ or } \lambda \neq \bar{\lambda})$, the accumulation levels of physical capital and human capital in competitive equilibrium can never be different from the planner's choice in opposite directions. Figure 1 illustrates physical and human capital in competitive equilibrium and the social optimum under different ranges of social weights for different values of the externality factor. It is clear to see that solid green line and the dashed red line for physical capital k and human capital h does not overlap each other for all different values of Δ.

[Insert Figure 1 here.]

3.2 Suboptimality of the equilibrium when the bequest motive is inoperative

This subsection analyzes the situation when the bequest motive is inoperative. In this environment, (6c) is not binding and thus, b=0. The *CE* in the steady state is characterized by (13a) and (13b), which is compared with (18a) and (18b) of the *SP* in the steady state. As we will see, the accumulation levels of physical and human capital in *CE* can be different from the *SP* in *opposite* directions for some values of the social weight.

First, note that without consumption externality (Δ=1), $\tilde{k}^{SP}(\bar{\lambda}) = \tilde{k}^{CE}(\bar{\lambda})$ at the laissez-faire supported social weight. Then, by comparing (13a) with (18a), it is clear that $\bar{\lambda} > \beta$.

Now, the bequest motive is inoperative. Lemma 1 remains valid. To see this, if we compare a parent's education investment condition in (9a) in *CE* with (18b) in *SP*, it is easy to verify that for all *t*, $\frac{\partial \tilde{k}^i}{\partial e^i} > 0$, *i*= *CE*, *SP*, and thus $\eta^i_{k,e} > \eta^i_{h,e}$ holds in the steady state. Moreover, it is easy to show that if a laissez-faire supported social weight $\bar{\lambda}$ exists, it must be unique. Therefore, Lemma 1 holds true here.

However, Lemma 2 no longer holds. If we compare the *CE* condition (13b) with the *SP* condition (18b), the equality results of $\tilde{k}^{SP}(\lambda^*) = \tilde{k}^{CE}$ and $e^{SP}(\lambda^*) = e^{CE}$ in Lemma 2 do not hold. Given that $\phi''(e) < 0$, from the comparison of (13b) with (18b), one can easily establish the following lemma.

Lemma 3. *For any Δ, if there exists a $\hat{\lambda}_1$ such that $k^{SP}(\hat{\lambda}_1) = \tilde{k}^{CE}$ holds, then $e^{SP}(\hat{\lambda}_1) > e^{CE}$; and if there exists a $\hat{\lambda}_2$ such that $e^{SP}(\hat{\lambda}_2) = e^{CE}$ holds, then $\tilde{k}^{SP}(\hat{\lambda}_2) < \tilde{k}^{CE}$. In addition, $\hat{\lambda}_1 > \lambda_2$.*

Now, as the bequest motive is inoperative, there exist $\hat{\lambda}_1 > \lambda_2$, such that $\tilde{k}^{SP}(\hat{\lambda}_2) < \tilde{k}^{CE}$ and $e^{SP}(\hat{\lambda}_1) > e^{CE}$. That is, under an inoperative bequest motive, the accumulation levels of physical and human capital in *CE* may be different from those in *SP* in different directions. To see this, we first start with the case $\lambda = \bar{\lambda}$, followed by the case $\lambda \neq \bar{\lambda}$.

Case 1. The social weight is equal to the laissez-faire supported social weight

With $\lambda = \bar{\lambda}$, first, when Δ>1, we have known $h^{SP}(\bar{\lambda}) > h^{CE}$, $\tilde{k}^{SP}(\bar{\lambda}) > \tilde{k}^{CE}$ and $k^{SP}(\bar{\lambda}) > k^{CE}$. Thus, like Proposition 1, the levels of physical and human capital in *CE* are different from those in *SP* in the same directions. Next, when Δ=1, we obtain $\tilde{k}^{SP}(\bar{\lambda}) = \tilde{k}^{CE}$ which, according to Lemma 3, implies $e^{SP}(\bar{\lambda}) > e^{CE}$, and thus $h^{SP}(\bar{\lambda}) > h^{CE}$. With $\tilde{k}^{SP}(\bar{\lambda}) = \tilde{k}^{CE}$ and $h^{SP}(\bar{\lambda}) > h^{CE}$, it follows that $k^{SP}(\bar{\lambda}) > k^{CE}$. Thus, when $\Delta \geq 1$, like Proposition 1, physical capital and human capital in *CE* are both under-accumulated as compared to the *SP*.

However, when Δ<1, then $\tilde{k}^{SP}(\bar{\lambda}) < \tilde{k}^{CE}$ does not imply $e^{SP}(\bar{\lambda}) < e^{CE}$, according to Lemma 3. Thus, it may lead to $h^{SP}(\bar{\lambda}) < h^{CE}$ or $h^{SP}(\bar{\lambda}) > h^{CE}$. Hence, it is not sure of whether human capital in

CE is under- or over-accumulated as compared to the *SP*. For a similar reason, it is not sure of whether physical capital in *CE* is under- or over-accumulated as compared to the *SP*. We summarize one of our two main findings as follows.

Proposition 2. *With an inoperative bequest motive and the social weight equal to the laissez-faire supported social weight* $\bar{\lambda}$, *if the consumption externality from the old dominates that from the middle-aged, the accumulation levels of physical and human capital in competitive equilibrium may differ from the social optimum in opposite directions.*

Case 2. **The social weight is different from the laissez-faire supported social weight**

Now, we proceed to the case $\lambda \neq \bar{\lambda}$. First, when Δ<1, according to Lemma 1, $\tilde{k}^{SP}(\bar{\lambda}) < \tilde{k}^{CE}$ and $h^{SP}(\bar{\lambda}) < h^{CE}$. Hence, there exist $\lambda_+^1 > \bar{\lambda}$ and $\lambda_+^2 > \bar{\lambda}$ such that $\tilde{k}^{SP}(\lambda_+^1) = \tilde{k}^{CE}$ and $h^{SP}(\lambda_+^2) = h^{CE}$. Moreover, according to Lemma 3, we know $h^{SP}(\lambda_+^1) > h^{CE}$ and $\tilde{k}^{SP}(\lambda_+^2) < \tilde{k}^{CE}$, which imply $k^{SP}(\lambda_+^1) > k^{CE} > k^{SP}(\lambda_+^2)$. Since k^{SP} is continuous in λ, there exists $\lambda_+^3 \in (\lambda_+^2, \lambda_+^1)$ such that $k^{SP}(\lambda_+^3) = k^{CE}$. As a result, we obtain (1) $h^{CE} < h^{SP}$ and $k^{CE} < k^{SP}$ for all $\lambda > \lambda_+^3$, and (2) $h^{CE} > h^{SP}$ and $k^{CE} > k^{SP}$ for all $\lambda < \lambda_+^2$, but (3) $h^{CE} < h^{SP}$ and $k^{CE} > k^{SP}$ for all $\lambda \in (\lambda_+^2, \lambda_+^3)$.

Similarly, when Δ>1, we follow the above procedure and can easily show that there exist λ_-^2 and λ_-^3 with $\lambda_-^2 < \lambda_-^3 < \bar{\lambda}$ such that (1) $h^{SP} > h^{CE}$ and $k^{SP} > k^{CE}$ for all $\lambda > \lambda_-^3$, and (2) $h^{SP} < h^{CE}$ and $k^{SP} < k^{CE}$ for all $\lambda < \lambda_-^2$, but (3) $h^{SP} > h^{CE}$ and $k^{SP} < k^{CE}$ for all $\lambda \in (\lambda_-^2, \lambda_-^3)$.

Finally, when Δ=1, one can easily show that the regimes of the social weight separating under- or over-accumulation of physical and human capital are like those when Δ>1, except now $\lambda_-^2 < \lambda_-^3 < \beta < \bar{\lambda}$. Thus, we obtain (1) $h^{SP} > h^{CE}$ and $k^{SP} > k^{CE}$ for all $\lambda > \lambda_-^3$, and (2) $h^{SP} < h^{CE}$ and $k^{SP} < k^{CE}$ for all $\lambda < \lambda_-^2$, but (3) $h^{SP} > h^{CE}$ and $k^{SP} < k^{CE}$ for all $\lambda \in (\lambda_-^2, \lambda_-^3)$.

Based on the above analysis, we construct the following corollary.

Corollary 1. *Assume that the bequest motive is inoperative.*

(i) *If the externality factor is greater than one, i.e.,* Δ>1, *there exist social weights* $\lambda_-^2 < \lambda_-^3 < \bar{\lambda}$ *such that, for any* $\lambda < (resp. >)\ \lambda_-^3$, *the CE over-accumulates (resp. under-accumulates) physical capital, but for any* $\lambda < (resp. >)\ \lambda_-^2$, *the CE over-accumulates (resp. under-accumulates) human capital.*

(ii) *If the externality factor is equal to one, i.e.,* Δ=1, *there exist social weights* $\lambda_-^2 < \lambda_-^3 < \beta < \bar{\lambda}$ *such that, for any*

$\lambda < (resp. >)\ \lambda_-^3$, *the* CE *over-accumulates (resp. under-accumulates) physical capital, but for any* $\lambda < (resp. >)\ \lambda_-^2$, *the* CE *over-accumulates (resp. under-accumulates) human capital.*

(iii) If the externality factor is less than one, i.e., $\Delta<1$*, there exist social weights* $\lambda_+^3 > \lambda_+^2 > \bar{\lambda}$ *such that, for any* $\lambda > (resp. <)\ \lambda_+^3$, *the* CE *under-accumulates (resp. over-accumulates) physical capital, but for any* $\lambda > (resp. <)\ \lambda_+^2$*, the* CE *under-accumulates (resp. over-accumulates) human capital.*

Figure 2 illustrates the range for the values of social weights under different values of the externality factor Δ wherein physical and human capital in *CE* differ from the *SP* in opposite directions. It is clear to see that, under the different values of Δ, there exists a range of moderate values of social weights such that the solid green line and the dashed red line for capital k and human capital h overlap each other. In particular, in *CE*, capital over-accumulates and human capital under-accumulates.

[Insert Figure 2 here.]

We summarize another main result in the following proposition.

Proposition 3. *With an inoperative bequest motive, for an externality factor that is larger than (resp. equal to, or smaller than) one, there exists a corresponding range of social weights different from the laissez-faire supported social weight, such that the accumulation levels of physical and human capital in* CE *are different from the* SP *in opposite directions.*

Thus, Propositions 2 and 3 together indicate that when the bequest motive is inoperative, for an externality factor, there exists values of the social weight, such that the accumulation levels of the two types of capital in competitive equilibrium differ from the planner's allocations in opposite directions.

4. Optimal Policy

In this section, we characterize the optimal policy in order to implement the *SP* allocation. We consider a tax policy consisting of an estate tax, a capital income tax, an education subsidy and a system of lump-sum taxes and subsidies. We follow Section 3 and maintain no population growth, and thus n=1.

Concerning the lump-sum taxes, we assume that working-aged agents in period t pay a lump-sum tax $T_t^m \geq 0.$ The revenue is devoted to financing a lump-sum transfer to the old $\kappa_t^o \geq 0$ in period t. Thus, one of the government constraints is

$$T_t^m - \kappa_t^o = 0. \tag{19a}$$

Next, working-aged agents also pay an estate tax rate τ_t^b on the inheritance they receive in period t and old agents pay a capital income tax rate τ_t^k on returns to savings in period t. These tax revenues are used to subsidize period-t education costs $\theta_t^e e_t$. Thus, another government budget constraint is

$$\tau_t^b b_t + \tau_t^k R_t s_{t-1} = \theta_t^e e_t. \tag{19b}$$

The budget constraints faced by an individual of generation t during the middle age of life in (4a) and the old age of life in (4b) are now modified, respectively, as

$$w_t h_t - T_t^m + (1-\tau_t^b) b_t = c_t + s_t + (1-\theta_t^e) e_t, \tag{20a}$$

$$(1-\tau_{t+1}^k) R_{t+1} s_t + \kappa_{t+1}^o = d_{t+1} + b_{t+1}. \tag{20b}$$

The optimization problem of the representative agent of generation t is to maximize (3) with respect to $\{c_t, d_{t+1}, e_t, b_{t+1}\}_{t=0}^{\infty}$ subject to (1), (20a) and (20b), given w_t and R_{t+1} for all t and given k_0 and h_0. The optimality conditions for s_t, e_t and b_{t+1} are as follows.

$$u_{\hat{c}_t}(\hat{c}_t, \hat{d}_{t+1}) = u_{\hat{d}_{t+1}}(\hat{c}_t, \hat{d}_{t+1})(1-\tau_{t+1}^k) R_{t+1}, \tag{21a}$$

$$(1-\theta_t^e) u_{\hat{c}_t}(\hat{c}_t, \hat{d}_{t+1}) = \beta u_{\hat{c}_{t+1}}(\hat{c}_{t+1}, \hat{d}_{t+2}) w_{t+1} \phi'(e_t) \tag{21b}$$

$$u_{\hat{d}_{t+1}}(\hat{c}_t, \hat{d}_{t+1}) \geq \beta u_{\hat{c}_{t+1}}(\hat{c}_{t+1}, \hat{d}_{t+2})(1-\tau_{t+1}^b), \tag{21c}$$

where the equality in (21c) holds if $b_{t+1}>0$.

Definition 3. For given h_0 and k_0 and a given path of tax policies $\{T_t^m, \kappa_t^o, \tau_t^b, \tau_t^k, \theta_t^e\}_{t=0}^{\infty}$, a competitive equilibrium is a path $\{c_t, d_t, k_t, s_t, e_t, b_t, h_t, w_t, R_t\}_{t=0}^{\infty}$ that satisfies firms' optimization (2a)-(2b), agents' optimization (21a)-(21c), the human capital accumulation (1), the goods market clearing condition (7), the government budget constraints (19a)-(19b), and the transversality condition $\lim_{t\to\infty} \beta^t u_{\hat{c}_t} b_t = 0$.

Let a value and a policy with a "crescent" symbol on their top as an optimal value and an optimal policy in competitive equilibrium. Then, the path of the optimal policy is $\left\{\widehat{T}_t^m, \widehat{\kappa}_t^o, \widehat{\tau}_t^b, \widehat{\tau}_t^k, \widehat{\theta}_t^e\right\}_{t=0}^{\infty}$. The optimal policy is such that the tax-corrected equilibrium path of $\left\{\widehat{c}_t, \widehat{d}_t, \widehat{k}_t, \widehat{e}_t\right\}$ coincides with the optimal path of $\left\{c_t^{SP}, d_t^{SP}, k_t^{SP}, e_t^{SP}\right\}$. By substituting savings from the budget constraint of the old in (4b) into the goods market condition in (7), the optimal path of bequest implied by the planner's solution is

$$b_t^{SP} = f'(\tilde{k}_t^{SP}) \tilde{k}_t^{SP} h_t^{SP} - d_t^{SP}. \tag{22}$$

If the optimal amount of bequest is positive, this implies that the bequest motive is operative along an equilibrium path. By contrast, a negative optimal amount of bequest implies that the bequest motive is inoperative along this equilibrium path. Then, mimicking this optimal path of bequest along the equilibrium path implies that the equilibrium bequest path attains the first best solution. To characterize the optimal tax policy, we distinguish two cases.

4.1 When the bequest motive is operative

In this case, the equilibrium path associated to the optimal policy is characterized by (14), (20a)-(20b) and (21a)-(21c). Note that, along with the use of (2a) and (2b), (21a)-(21c) lead to, respectively,

$$\frac{u_{\hat{c}_t}}{u_{\hat{d}_{t+1}}} = (1-\hat{\tau}_{t+1}^k) f'(\tilde{k}_{t+1}), \tag{23a}$$

$$\frac{u_{\hat{c}_t}}{u_{\hat{c}_{t+1}}} = \beta \frac{1}{1-\hat{\theta}_t^e}\left[f(\tilde{k}_{t+1}) - \tilde{k}_{t+1} f'(\tilde{k}_{t+1}) \right] \phi'(e_t), \tag{23b}$$

$$\frac{u_{\hat{c}_t}}{u_{\hat{d}_t}} = \frac{1}{\beta(1-\hat{\tau}_t^b)}. \tag{23c}$$

To investigate which policy instruments should be used in order to achieve a social optimum in a market economy, we compare social optimum conditions (16), (17a) and (17b) with tax-corrected competitive equilibrium conditions (23c), (23a) and (23b), respectively, along with the use of (17c). We obtain the following result.

Proposition 4. *When the bequest motive is operative, the optimal taxes are* $\hat{\tau}_t^k = 1-\Delta$, $\hat{\theta}_t^e = 1-\frac{\beta}{\lambda}$, *and* $\hat{\tau}_t^b = 1-\frac{1}{\Delta}\frac{\lambda}{\beta}$.

When the bequest motive is operative, consumption externalities are the only source of suboptimality. In this case, consumption externalities and altruism affect decisions on savings, education, and bequests. Thus, the optimal policy depends on consumption externalities and altruism. Yet, lump-sum taxes are irrelevant due to Ricardian equivalence.

First, the sign of optimal capital taxes $\hat{\tau}_t^k$ depends only on the externality factor Δ. The optimal capital tax is positive if the externality factor is smaller than unity,[18] but is negative if otherwise. Intuitively, if Δ<1 and thus, the consumption externality coming from the working-aged is weaker than that coming from the old, the working-aged save too much and consume too little. Then, a positive capital tax serves to lower their savings. By contrast, if the consumption externality coming from the old is stronger, the working-aged save too little and consume too much. Then, a subsidy to capital serves to increase the savings of the working-aged.

Next, the education subsidy $\hat{\theta}_t^e$ is not affected by consumption externalities but is affected by the

[18] Namely, if the externality factor Δ > (*resp.* <) 1; i.e., if $\sigma_c + \varphi_c > (resp. <)\ \sigma_d + \varphi_d$.

altruism motive. The education subsidy is positive if the altruism factor β is smaller than the social weight λ, and is negative if otherwise. Intuitively, education is a parent's transfer in kind to children. If the altruism factor is smaller than the social weight, parental education investment for children is less than what arranged by the social planer. Hence, an education subsidy is optimal. Conversely, if the altruism factor is larger than the social weight, parental education investment for children is more than what devised by the social planer. Thus, an education tax is optimal.

Finally, a bequest is a mix of savings and transfer in goods. The sign of the bequest tax $\hat{\tau}_t^b$ depends not only on consumption externalities Δ but also on the altruism motive. If Δ=1, the effects of consumption externalities from the working-aged completely offset those from the old. Then, only the altruism (i.e., transfer) motive is at work, and the altruism factor relative to the social weight is what matters for the sign of a bequest tax. In this case, if the altruism factor β is smaller than the social weight λ, the old leave bequests less than the optimum, and a subsidy to bequests is optimal; if otherwise, a tax on bequests is optimal. Alternatively, if $\Delta \neq 1$, consumption externalities are also at work. Then, if the altruism-factor discounted externality factor $\beta\Delta$ is larger than the social weight λ, the old leave bequests more than the optimum, and a bequest tax is optimal; if otherwise, a subsidy to bequests is optimal.

4.2 When the bequest motive is inoperative

In this case, (21c) is not binding and b_t=0. The equilibrium path associated to the optimal policy is characterized by (14), (20a)-(20b), (21a)-(21b), and b_t=0. The optimal policy is then analyzed by comparing (17a) and (17b) with (21a) and (21b), respectively. With the use of (17c), we get the optimal capital tax rate $\hat{\tau}_t^k = 1 - \Delta$ and the education subsidy rate $\hat{\theta}_t^e = 1 - \frac{\beta}{\lambda}$, which are the same as in the case when the bequest motive is operative.

The suboptimality due to the inoperativeness of the bequest motive is resolved through lump-sum taxes. To obtain the optimal taxes due to inoperative bequest motive, we substitute the period t government budget constraint in (19b) into the period t old-age individual's budget constraint in (20b), with the use of the rate of returns $R_t = f'(\tilde{k}_t)$ in (2a) and the capital accumulation $s_{t-1} = k_t = \tilde{k}_t h_t$ in (7), along with n=1 and b_t=0. Then, we obtain consumption of the old in period t as follows.

$$\hat{d}_t = f'(\tilde{k}_t)\tilde{k}_t\hat{h}_t - \hat{\theta}_t^e\hat{e}_t + \hat{\kappa}_t^o. \tag{24}$$

In order for consumption, capital and human capital in the tax-corrected market equilibrium condition in (24) to replicate the social optimum in (22),[19] the lump-sum transfer to the old in period t

[19] Note that (22) can be rewritten as $d_t^{SP} = f'(\tilde{k}_t^{SP})\tilde{k}_t^{SP}h_t^{SP} - b_t^{SP}$.

is set at $\widehat{\kappa}_t^o = \widehat{\theta}_t^e \widehat{e}_t - b_t^{SP}$. By using (19a), it follows that the lump-sum transfer to the old is paid by the lump-sum tax to the working-aged in period *t* and thus, $\widehat{T}_t^m = \widehat{\theta}_t^e \widehat{e}_t - b_t^{SP}$. The optimal taxes are as follows.

Proposition 5. *When the bequest motive is inoperative, the optimal taxes are* $\widehat{\tau}_t^k = 1-\Delta$, $\widehat{\theta}_t^e = 1-\frac{\beta}{\lambda}$, *and* $\widehat{\kappa}_t^o = \widehat{T}_t^m = \widehat{\theta}_t^e \widehat{e}_t - b_t^{SP}$.

Thus, even though the bequest motive is inoperative, capital taxes and education subsidies are the same as those in the case when the bequest motive is operative, because consumption externalities and transfer motives remain at work. Now, the amount of bequest is zero in equilibrium and thus, a bequest tax or a bequest subsidy is not feasible. However, the optimal bequest desired by the social planner in (22) is negative, $b_t^{SP} < 0$. In this case, the social optimum calls for intergenerational lump-sum transfers.

Note that $b_t^{SP} < 0$ indicates inoperative bequest motives, wherein the altruism factor β is less than the threshold $\overline{\beta}$. First, if the altruism factor β is smaller than the social weight λ, $\widehat{\theta}_t^e > 0$ and thus, education is subsidized at the amount $\widehat{\theta}_t^e \widehat{e}_t > 0$. In this case, it is optimal to subsidize retirees a lump sum equal to the difference between the education subsidy and the optimal bequest $\widehat{\kappa}_t^o = \widehat{\theta}_t^e \widehat{e}_t - b_t^{SP} > 0$ and to tax the working-aged the same lump sum. Alternatively, if the altruism factor β is larger than the social weight λ, $\widehat{\theta}_t^e < 0$ and thus, education is taxed at the amount $-\widehat{\theta}_t^e \widehat{e}_t > 0$. In this case, it is still optimal to subsidize retirees a lump sum $\widehat{\kappa}_t^o = \widehat{\theta}_t^e \widehat{e}_t - b_t^{SP} > 0$ and tax the working-aged the same lump sum if the absolute value of optimal bequests desired by the social planner $-b_t^{SP}$ is larger than the amount of education taxes $-\widehat{\theta}_t^e \widehat{e}_t$, and it is optimal to tax retirees a lump sum and subsidize the working-aged the same lump sum if otherwise.

5. Concluding Remarks

This paper studies suboptimality of physical and human capital accumulation in a three-period OLG model with consumption externalities and altruism. In the model, parents have willingness to finance education and leave bequests for children.

We find that parents are always willing to finance children's education, but the motive to leave bequests may be operative or inoperative. The mechanism leading to suboptimality in our model depends on consumption externalities and altruism. When the bequest motive is operative, the lack of optimality is due to consumption externalities. Then, like the existing model with consumption externalities but without parental altruism, there is either over-accumulation or under-accumulation of both physical and human capitals. However, when the bequest motive is inoperative, suboptimality is due to consumption externalities and inoperative bequest motives. In this situation, for a range in the value of the social weight, physical capital is over-accumulated but human capital is under-accumulated in equilibrium, which lends support to the policy of implementing education subsidies and public pensions at the same time, a policy prevailing in OECD countries.

Optimal tax policies are examined in order for the equilibrium to attain the first best solution. Capital is taxed or subsidized, depending on the externality factor being smaller or larger than unity, and education is subsidized or taxed depending on the altruism factor being smaller or larger than the social weight. Only under operative bequest motives, bequests are taxed or subsidized, depending upon the altruism-factor discounted externality factor being larger or smaller than the social weight. By contrast, if bequest motives are inoperative, there is no bequest in equilibrium, but the optimal bequest is negative. Then, the social optimum calls for intergenerational lump-sum transfers from the working-aged to retirees when the education is subsidized or taxed at an amount smaller than the absolute value of optimal bequests, and calls for intergenerational lump-sum transfers from the retirees to the working-aged if otherwise.

References

Abel, AB. (1985) Precautionary saving and accidental bequests. *American Economic Review* 75, 777–791.

Abel, A.B. (1987) Operative gift and bequest motives. *American Economic Review* 77, 1037-1047.

Abel, A.B. (1988) An analysis of fiscal policy under operative and inoperative bequest motives. In E. Helpman, A. Razin and E. Sadka (eds.), *Economic Effects of the Government Budget*, M.I.T. Press, 91-106.

Abel, A.B. (2005) Optimal taxation when consumers have endogenous benchmark levels of consumption. *Review of Economic Studies* 75, 21-42.

Abel, A.B. and M. Warshawsky. (1988) Specification of the joy of giving: insights from altruism. *Review of Economics and Statistics* 70, 145-149.

Alonso-Carrera, J., J. Caballé and X. Raurich (2007) Aspirations, habit formation and bequest motive. *Economic Journal* 117, 813-836

Alonso-Carrera, J., J. Caballé and X. Raurich (2008) Estate taxes, consumption externalities, and altruism. *Journal of Public Economics* 92, 1751–1764.

Andreoni, J. (1990) Impure altruism and donations to public goods: a theory of warm-glow giving. *Economic Journal* 100, 464-477.

Barnett, R.C., J. Bhattacharya and H. Bunzel (2013) Deviant generations, Ricardian equivalence, and growth cycles. *Economic Theory* 52, 367–396.

Barro, R.J. (1974) Are government bonds net wealth? *Journal of Political Economy* 82, 1095-1117.

Barro, R.J. and J.-W. Lee (1993) International comparisons of educational attainment. *Journal of Menetary Economics* 32, 363–394.

Bernheim, B., A. Schleifer and L. Summers, L. (1985) The strategic bequest motive. *Journal of Political Economy* 93, 1045–1075.

Bishnu, M. (2013) Linking consumption externalities with optimal accumulation of human and physical capital and intergenerational transfers. *Journal of Economic Theory* 148, 720–742.

Boldrin, M. and A. Montes (2005) The intergenerational state education and pensions. *Review of Economic Studies* 72, 651-664.

Buiter, W.H. (1979) Government finance in an overlapping-generations model with gifts and bequests. In G.M. von Furstenberg (ed.), *Social Security Versus Private Saving*, Cambridge: Ballinger.

Brown, P.H. (2006) Parental education and investment in children's human capital in rural China, *Economic Development and Cultural Change* 54, 759-789.

Card, D. (1999). The causal effect of education on earnings. *Handbook of Labor Economics* 3A, North-Holland, Amsterdam.

Carmichael, J. (1982) On Barro's theorem of debt neutrality: the irrelevance of net wealth. *American Economic Review* 72, 202-213.

Ciccone, A. and G. Perri (2006) Identifying human-capital externalities: theory with applications. *Review of Economic Studies* 73, 381–412.

Clark, A.E. and A.J. Oswald (1996) Satisfaction and comparison income. *Journal of Public Economics* 61, 359–381.

Diamond, P.A. (1965) National debt in a neoclassical growth model. *American Economic Review* 55, 1126-1250.

Docquier, F., O. Paddison and P. Pestieau (2007) Optimal accumulation in an endogenous growth setting with human capital. *Journal of Economic Theory* 134, 361-378.

Dusenbery, J.S. (1949) *Income, Saving, and the Theory of Consumer Behavior*. New York: Oxford University Press.

Gali, J. (1994) Keeping up with Joneses: consumption externalities, portfolio choice and asset price. *Journal of Money, Credit and Banking* 26, 1-8.

Gertler, P. and P. Glewwe (1990) The willingness to pay for education in developing countries: evidence from rural Peru. *Journal of Public Economics* 42, 251-275.

Kornrich, S. and F. Furstenberg (2013) Investing in Children: Changes in Parental Spending on Children, 1972–2007, *Demography* 50, 1-23.

Lange, F. and R. Topel (2006) The social value of education. *Handbook of the Economics of Education*, 459-509, Elsevier, Amsterdam.

Ihori, T., K. Kamada and T. Sato (2017) Altruism, liquidity constraint, and investment in education. *Journal of Public Economic Theory* 19, 409-425.

Levine, R. and Renelt (1992) A sensitivity analysis of cross-country growth regressions. *American Economic Review* 82, 942-963.

Lillard, L.A. and R.J. Willis (1997) Motives for intergenerational transfers: evidence from Malaysia, *Demography* 34, 115-134

Ljungqvist, L. and H. Uhlig (2000) Tax policy and aggregate demand management under catching up with Joneses. *American Economic Review* 90, 356-366.

Lucas, R.E., Jr. (1988) On the mechanics of economic development. *Journal of Monetary Economics* 22, 3-42.

Luttrmer, E.F.P (2005) Neighbors as negatives: relative earning and well-being. *Quarterly Journal of Economics* 120, 963–1002.

Michel, P., E. Thibault and J. Vidal (2006) Intergenerational altruism and neoclassical growth models. In S. Kolm and J.M. Ythier (eds.) *Handbook on the Economics of Giving, Reciprocity and Altruism*, 1055–1106, Elsevier, Amsterdam.

Miller, E.I. (1997) Parents' views on the value of a college education and how they will pay for it, *Journal of Student Financial Aid* 27, 7-20.

Page, B.R. (2003) Bequest taxes, inter vivos gifts, and the bequest motive. *Journal of Public Economics* 87, 1219–1229.

Romer, P. (1990) Endogenous technical change. *Journal of Political Economy* 98, S71-S102.

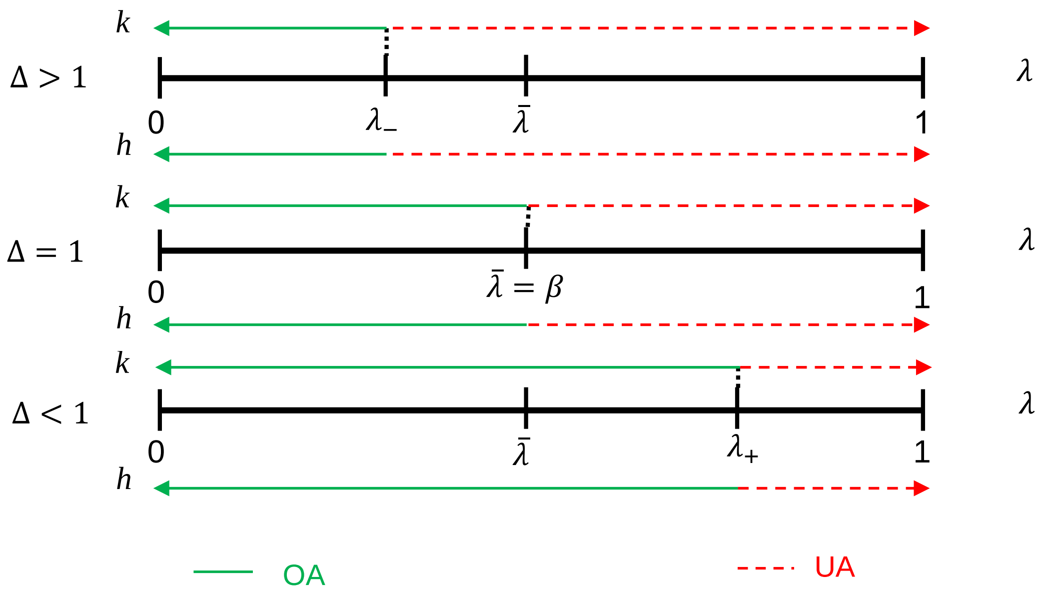
Rudd, J. (2000) Empirical evidence on human capital spillovers. *Finance and Economics Discussion Series* 2000-46, Board of Governors of the Federal Reserve System, Washington.

Steelman, L.C. and B. Powell (1991) Sponsoring the next generation: parental willingness to pay for higher education. *American Journal of Sociology* 96, 1505-1529.

Veblen, T. (1912) *The Theory of the Leisure Class*. New York: Macmillan.

Weil, P. (1987) Love the children: reflections on the Barro debt neutrality theorem. *Journal of Monetary Economics* 19, 377-391.

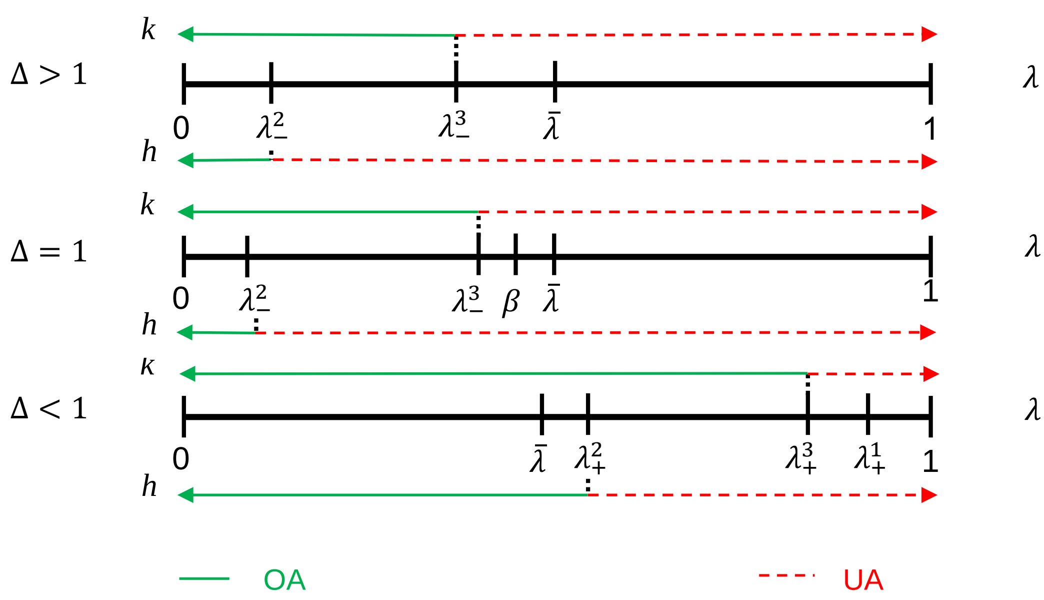
Yamarik, S.J. (2008) Estimating returns to schooling from state-level data: a macro-Mincerian approach. *B.E. Journal of Macroeconomics* 8, 747–752.



Note: OA: Over-accumulation; UA: Under-accumulation.

Figure 1. Over- or under-accumulation of both types of capital when the bequest motive is operative.

.



Note: OA: Over-accumulation; UA: Under-accumulation.

Figure 2. Over- or under-accumulation of both types of capital when the bequest motive is inoperative.