Productivity Investment, Power Law, and Welfare Gains from Trade

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Why Do We Care About Productivity Distributions?

Melitz (2003): Firm selection matters for gains from trade.

Quantifying gains from trade.

But:

1. Exogenously assumes productivity distribution.
   Specification on productivity matters: Bee and Schiavo (2015), and Nigai (2017).
2. Truncation to productivity distribution due to firm selection.
   Because weak firms die out...

Motivation

Productivity is a result of R&D and investment activities!

Why does the empirical distribution exhibits power law / Pareto tail?

How does productivity distribution respond to trade liberalization?

What is the implication on welfare gains from trade?

Our Model

1. Incorporates firm-level productivity investment decision. Sutton (1991)
2. Heterogeneous investment efficiency (talent / entrepreneurship).

Our Results

The productivity distribution always has a Pareto tail.

Requires almost no assumptions on the distribution of talent.

Robust against :

1. Investment cost function (a subclass of smoothly varying function).
2. Demand system (asymptotic CES).

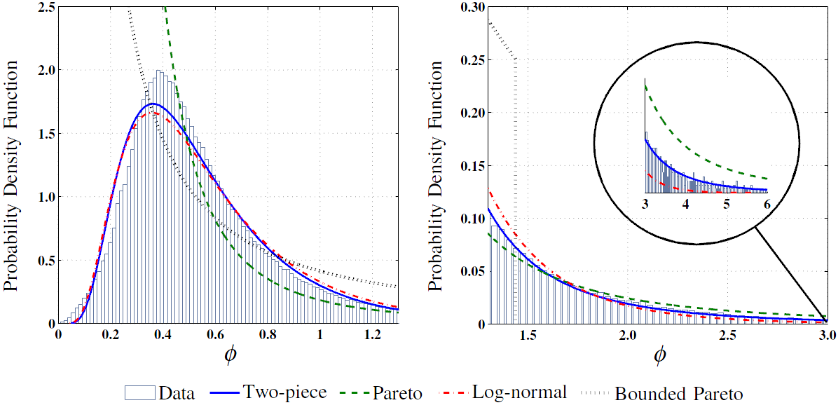
Our Results

Intensive margin matters so it's more than a specification issue.

Trade liberalization results in:

1. Extensive margin: more firm selection.
2. Intensive margin:
   Exporters invest more.
   Non-Exporters invest less (e.g. Pavcnik (2002), Fernandes (2007), and Baldwin and Gu (2009)).
3. New gains from trade through variable trade cost.
4. Less welfare elasticity.

Exogenous Distribution



Source: Figure 4 of Nigai (2017)

Exogenous Distribution

Axtell (2002): Power law to the right tail.

Pareto: Chaney (2008), Melitz and Redding (2015), and hundreds of studies.

Lognormal: Eeckhout (2004), Head et al. (2014)

Bee et al. (2017): Neither Pareto nor lognormal!

Nigai (2017, JIE): mixed distribution.

Exogenous Distribution

The distribution is exogenously assumed!

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(This is important so must be repeated by three times)

Endogenous Productivity

Binary technology choice: Yeaple (2005).

Matching between firms and workers: Monte (2011) and Sampson (2014)

Sutton-way: Bas and Ledezma (2015).

Endogenous Productivity

Power law is not addressed.

Gains from trade is not examined.

Bas and Ledezma (2015): the effect of trade liberalization on exporter is ambiguous.

Welfare Gains from Trade

Arkolakis et al. (2012):

(R1) Trade balances.

(R2) Constant ratio between aggregate profit and revenue.

(R3) Constant bilateral trade elasticity $d \ln(\lambda_x/\lambda_0)/d \ln \tau$ for all x countries.

λ_i denotes the expenditure on products of country i in the domestic country 0.

If (R1)-(R3) holds, then welfare depends on λ_0 only.

Krugman model and Melitz model alike!

Consumer

Symmetric preference and income.

Utility for consuming each variety υ: $U = \int_{\upsilon \in \Upsilon} u\left(q\left(\upsilon\right)\right) d\upsilon$.

Implied demand per variety: $p\left(\upsilon\right) = D\left(q\left(\upsilon\right); A\right)$.

A is endogenously determined.

Producer

Monopolistic competitive firms.

Labor is the only input, and is considered as numeraire.

Entry cost: κ_e.

Production cost: $q/\varphi + \kappa_D$.

Productivity φ is endogenously determined through investment.

Investment

Investment function:

$$\varphi = B\left(t \cdot k\right).$$

Labor input k.

Talent / entrepreneurship $t \in (t_L, \infty)$ with $t_L \geq 0$.

$B'\left(t \cdot k\right) > 0,\ B''\left(t \cdot k\right) < 0.$

For convenience, the cost of investment is:

$$k = \frac{B^{-1}\left(\varphi\right)}{t} \equiv \frac{V\left(\varphi\right)}{t} \equiv \gamma V\left(\varphi\right).$$

The talent index $\gamma \in (0, \gamma_H)$ follows a distribution with p.d.f. $f\left(\gamma\right)$.

Basic Setting

Total Profit:

$$\Pi(\varphi) = \pi(\varphi) - \gamma V(\varphi),$$

$$\pi(\varphi) = pq - \varphi^{-1} q - \kappa_D.$$

Timing:

1. Entry Stage: Each firm pays κ_e to enter, and then observes γ respectively.
2. Investment Stage: Each firm decides whether to invest, and if yes, the level of φ.
3. Production Stage: Each firm decides whether to produce, and if yes, the price of its variety.

Preliminary Example

CES demand: $q = A^{\frac{1}{\sigma}} p^{-\frac{1}{\sigma}}$.

Power function: $k(\varphi) = \gamma\varphi^{\beta}$

Optimal output: $q(\varphi) = A\rho^{\sigma}\varphi^{\sigma}$, where $\rho \equiv (\sigma - 1)/\varphi$ and $A \equiv L/P^{1-\sigma}$.

Investment Stage: each firm solves

$$\max_{\varphi} \Pi(\varphi) = \frac{A\rho^{\sigma}}{\sigma - 1}\varphi^{\sigma-1} - \gamma\varphi^{\beta}.$$

Optimal productivity:

$$\widetilde{\varphi}(\gamma) = \frac{A\rho^{\sigma}}{\beta}^{\frac{1}{\theta}} \gamma^{-\frac{1}{\theta}},$$

where $\theta \equiv \beta - \sigma + 1 > 0$ must hold to ensure the existence of optimality.

Preliminary Example

Zero cutoff profit condition (ZCP):

$$\Pi\left(\widetilde{\varphi}\left(\gamma\right);\gamma\right) \geq 0 \text{ if and only if } \gamma \leq \gamma_D.$$

Entry Stage: the free entry condition

$$\int_0^{\gamma_D} \Pi\left(\widetilde{\varphi}\left(\gamma\right);\gamma\right) dF\left(\gamma\right) = \kappa_e$$

pins down A along with ZCP and $\widetilde{\varphi}\left(\gamma\right)$.

Preliminary Example

Productivity distribution:

$$g(\varphi) = \frac{f(\gamma(\varphi))}{F(\gamma_D)} A \left(\frac{\rho^{\sigma}}{\beta}\right) \theta \varphi^{-\theta-1}.$$

Note that $\frac{\partial \gamma(\varphi)}{\partial \varphi} < 0$ and $\lim_{\gamma \to 0} \varphi(\gamma) = \infty$.

A, σ, β, ρ, θ, γ_D are all independent of φ.

If $\lim\limits_{\gamma \to 0} f(\gamma) = K > 0$, then

$$g(\varphi) \approx \frac{K}{F(\gamma_D)} A \left(\frac{\rho^{\sigma}}{\beta}\right) \theta \varphi^{-\theta-1}.$$

Preliminary Example

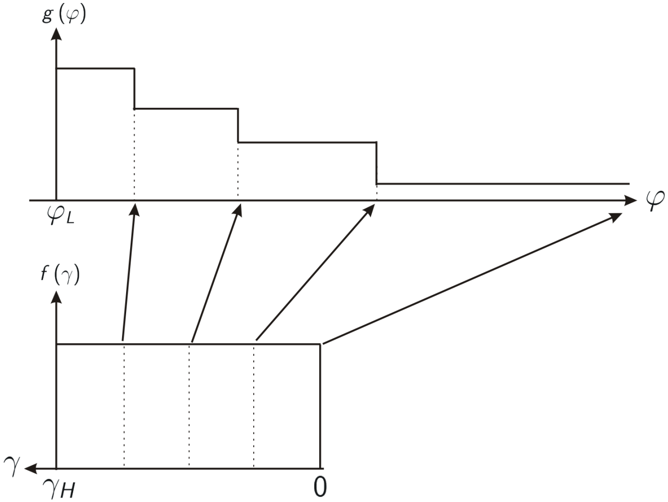
1. Since $\lim_{\gamma \to 0} f(\gamma) = K$, we can express $f(\gamma)$ as $f(\gamma) = K + h(\gamma)$, where $\lim_{\gamma \to 0} h(\gamma) = 0$.
2. Therefore, $g(\varphi) = A\left(\frac{\rho^\sigma}{\beta}\right)\theta K \varphi^{-\theta-1} + A\left(\frac{\rho^\sigma}{\beta}\right)\theta h(\gamma)\varphi^{-\theta-1}$.
3. Clearly, $\lim_{\varphi \to \infty} \varphi^{-\theta-1} = 0$, $\lim_{\varphi \to \infty} h(\gamma(\varphi)) = 0$, and $\lim_{\varphi \to \infty} h(\gamma(\varphi))\varphi^{-\theta-1} = 0$.
4. Since

$$\lim_{\varphi \to \infty} \frac{h(\gamma(\varphi))\varphi^{-\theta-1}}{\varphi^{-\theta-1}} = \lim_{\varphi \to \infty} h(\gamma(\varphi)) = 0,$$

it implies that the rate of convergence of $h(\gamma(\varphi))\varphi^{-\theta-1}$ dominates that of $\varphi^{-\theta-1}$. Thus, there is a φ_0 where for all $\varphi \geq \varphi_0$

$$g(\varphi) \approx \frac{K}{F(\gamma_D)} A\left(\frac{\rho^\sigma}{\beta}\right)\theta\varphi^{-\theta-1}.$$

If $\gamma \sim U[0, \gamma_H]$ and $\kappa_D = 0$



Smooth Variation

Definition

Definition 1. A function $v(x)$ is a *regularly varying function* if and only if $v(x)$ can be expressed as

$$v(x) = x^{\alpha} l(x),$$

where $l(x)$ is a slowly varying function, i.e., for any $\lambda > 1$,

$$\lim_{x \to \infty} \frac{l(\lambda x)}{l(x)} = 1.$$

Definition

Definition 2. A *Smoothly Varying Function* is a infinitely differentiable regularly varying function $v(x)$, such that for all $n \geq 1$

$$\lim_{x \to \infty} \frac{x^n v^{(n)}(x)}{v(x)} = \beta(\beta - 1) \ldots (\beta - n + 1),$$

where $v^{(n)}(x)$ denotes for the n-th derivative of $v(x)$.

Power Law of Productivity

Assumption

Assumption 1. *The inverse demand for each variety is a smoothly varying function* $p = D(q;A) \equiv q^{-\frac{1}{\sigma}} Q(q;A)$*, where* $\sigma > 1$ *and* $\lim_{q\to\infty} Q(q;A) = C_Q > 0$*. The investment cost is a smoothly varying function* $k(\varphi) = \gamma V(\varphi) \equiv \gamma \varphi^{\beta} L(\varphi)$*, where* $\beta > 1$ *and* $\lim_{\varphi\to\infty} L(\varphi) = C_L > 0$*.*

Proposition

Proposition 1. *Under Assumption 1, suppose that*

$$\lim_{\gamma\to 0} f(\gamma) = K > 0,$$

and $\theta \equiv \beta + 1 - \sigma > 0$*, the productivity distribution is approximately*

$$g(\varphi) \approx \frac{K}{F(\gamma_D)} \frac{C_Q^{\sigma}}{C_L} \rho^{\sigma} \frac{\theta}{\beta} \varphi^{-\theta-1}.$$

Why Smooth Variation?

Because smoothly varying functions are general!

For example, polynomial functions are smoothly varying.

Demand systems that are asymptotically CES are widely applied.

Demand Class	Inverse Demand Function	C_Q
CES	$p(q) = q^{-\frac{1}{\sigma}} A^{\frac{1}{\sigma}}$	$A^{\frac{1}{\sigma}}$
CREMR	$p(q) = q^{-\frac{1}{\sigma}} A^{\frac{1}{\sigma}} \left(1 - \omega q^{-1}\right)^{\frac{\sigma-1}{\sigma}}$ where $q > \sigma\omega$	$A^{\frac{1}{\sigma}}$
CEMR	$p(q) = q^{-\frac{1}{\sigma}} \left(A^{\frac{1}{\sigma}} + \alpha q^{-\frac{\sigma-1}{\sigma}}\right)$	$A^{\frac{1}{\sigma}}$
Bipower inverse	$p(q) = q^{-\frac{1}{\sigma}} A^{\frac{1}{\sigma}} \left(1 + \widehat{A}^{\frac{1}{\zeta}} A^{-\frac{1}{\sigma}} q^{\frac{1}{\sigma}-\frac{1}{\zeta}}\right)$ where $\sigma > \zeta$	$A^{\frac{1}{\sigma}}$
Pollak	$p(q) = q^{-\frac{1}{\sigma}} A^{\frac{1}{\sigma}} \left(1 - \widehat{A} q^{-1}\right)^{\frac{1}{\sigma}}$	$A^{\frac{1}{\sigma}}$

Table: Smoothly Varying Inverse Demand

Why Smooth Variation?

Because smoothly varying function can approximate the tail behaviors of regularly varying functions.

Theorem

(Theorem 1.8.2, Bingham et al. (1989)) *For a regularly varying function f, there exists smoothly varying functions f_1 and f_2, with $f_1 \sim f_2$ and $f_1 \leq f \leq f_2$ on some neighbourhood of infinity. In particular, for a regularly varying function f there exists a smoothly varying function g such that $g \sim f$.*

Power Law of Firm Size

Let $s = pq$

Corollary

Corollary 1. *Under Assumption 1, suppose that*

$$\lim_{\gamma \to 0} f(\gamma) = K > 0,$$

and $\theta \equiv \beta + 1 - \sigma > 0$, *the distribution of firm size* s *follows the power law with a tail index* $\frac{\theta}{\sigma - 1}$, *i.e.,*

$$\lim_{s \to \infty} g(s) \approx \frac{K}{F(\gamma_D)} \frac{C_Q^{\frac{\beta\sigma}{\sigma-1}}}{C_L} \left(\frac{\sigma - 1}{\sigma} \right)^{\beta} \frac{\theta}{\beta\sigma} s^{-\frac{\theta}{\sigma-1} - 1}.$$

Setting

CES utility.

Power function investment cost.

Total cost of exporting: $\tau q/\varphi + \kappa_X$.

Timing:

1. Entry Stage.
2. Investment Stage.
3. Production Stage: each firm can further decides whether to export and the price to charge.

Optimality

Production optimality implies that

$$\pi_D(\varphi) = \frac{A\rho^\sigma}{\sigma - 1}\varphi^{\sigma-1} - \kappa_D,$$

$$\pi_X(\varphi) = \tau^{1-\sigma}\frac{A\rho^\sigma}{\sigma - 1}\varphi^{\sigma-1} - \kappa_X,$$

where

$$A \equiv L/P^{1-\sigma}.$$

Optimal Investment

Profit for non-exporters and exporters:

$$\Pi_D(\varphi) = \pi_D(\varphi) - \gamma\varphi^{\beta},$$
$$\Pi_X(\varphi) = \pi_D(\varphi) + \pi_X(\varphi) - \gamma\varphi^{\beta}.$$

Optimal Productivity

$$\varphi = \begin{cases} A^{\frac{1}{\theta}} \left(\frac{\rho^{\sigma}}{\beta}\right)^{\frac{1}{\theta}} \gamma^{-\frac{1}{\theta}} & \text{for non-exporting firms,} \\ \left(1+\tau^{1-\sigma}\right)^{\frac{1}{\theta}} A^{\frac{1}{\theta}} \left(\frac{\rho^{\sigma}}{\beta}\right)^{\frac{1}{\theta}} \gamma^{-\frac{1}{\theta}} & \text{for exporting firms.} \end{cases}$$

Zero Cutoff Profit Conditions

Firms must not make negative profits: $\Pi_D(\gamma) \geq 0$ and $\Pi_X(\gamma) \geq \Pi_D(\gamma)$.

Therefore:

$$\gamma_D \equiv \left[\kappa_D^{-1} A^{\frac{\beta}{\theta}} \left(\frac{\rho^\sigma}{\beta}\right)^{\frac{\beta}{\theta}} \left(\frac{\beta}{\sigma-1} - 1\right)\right]^{\frac{\theta}{\sigma-1}},$$

$$\gamma_X \equiv \left[\kappa_X^{-1} \left[\left(1+\tau^{1-\sigma}\right)^{\frac{\beta}{\theta}} - 1\right] A^{\frac{\beta}{\theta}} \left(\frac{\rho^\sigma}{\beta}\right)^{\frac{\beta}{\theta}} \left(\frac{\beta}{\sigma-1} - 1\right)\right]^{\frac{\theta}{\sigma-1}}.$$

Zero Cutoff Profit Conditions

Assumption

Assumption 2. *Assume that*

$$\frac{\gamma_X}{\gamma_D} \equiv \delta \equiv \left(\frac{\kappa_D}{\kappa_X}\right)^{\frac{\theta}{\sigma-1}} \left[\left(1+\tau^{1-\sigma}\right)^{\frac{\beta}{\theta}} - 1\right]^{\frac{\theta}{\sigma-1}} < 1,$$

i.e., the fixed exporting cost κ_X must be large enough.

This means that $\kappa_X > \kappa_D$.

Otherwise, all firms are exporters.

Free Entry

Firms are subjected to free entry

$$\overline{\pi} = \kappa_e,$$

where

$$\overline{\pi} = \int_0^{\gamma_X} \Pi_X(\gamma)\, dF(\gamma) + \int_{\gamma_X}^{\gamma_D} \Pi_D(\gamma)\, dF(\gamma).$$

The aggregate price relates mass of entrants with $A \equiv L/P^{1-\sigma}$.

$$\begin{aligned} P^{1-\sigma} =& M_e \left[\int_{\gamma_X}^{\gamma_D} \rho^{\sigma-1} \varphi(\gamma)^{\sigma-1}\, dF(\gamma) + \int_0^{\gamma_X} \rho^{\sigma-1} \varphi(\gamma)^{\sigma-1}\, dF(\gamma) \right] \\ &+ M_e \int_0^{\gamma_X} \tau^{1-\sigma} \rho^{\sigma-1} \varphi(\gamma)^{\sigma-1}\, dF(\gamma). \end{aligned}$$

Equilibrium Productivity

Equilibrium productivity:

$$\varphi(\gamma) = \begin{cases} \kappa_D^{\frac{1}{\beta}} \gamma_D^{\frac{\sigma-1}{\beta\theta}} \left(\frac{\beta}{\sigma-1} - 1\right)^{-\frac{1}{\beta}} \gamma^{-\frac{1}{\theta}} & \text{if } \gamma \in (\gamma_X, \gamma_D] \\ \left(1+\tau^{1-\sigma}\right)^{\frac{1}{\theta}} \kappa_D^{\frac{1}{\beta}} \gamma_D^{\frac{\sigma-1}{\beta\theta}} \left(\frac{\beta}{\sigma-1} - 1\right)^{-\frac{1}{\beta}} \gamma^{-\frac{1}{\theta}} & \text{if } \gamma \in [0, \gamma_X] \end{cases}$$

Let $\varphi_D \equiv \kappa_D^{\frac{1}{\beta}} \gamma_D^{\frac{\sigma-1}{\beta\theta}} \left(\frac{\beta}{\sigma-1} - 1\right)^{-\frac{1}{\beta}} \gamma_D^{-\frac{1}{\theta}}$,

$\varphi_{DX} \equiv \left(1+\tau^{1-\sigma}\right)^{\frac{1}{\theta}} \kappa_D^{\frac{1}{\beta}} \gamma_D^{\frac{\sigma-1}{\beta\theta}} \left(\frac{\beta}{\sigma-1} - 1\right)^{-\frac{1}{\beta}} \gamma_D^{-\frac{1}{\theta}}$, and

$\varphi_X \equiv \left(1+\tau^{1-\sigma}\right)^{\frac{1}{\theta}} \kappa_D^{\frac{1}{\beta}} \gamma_D^{\frac{\sigma-1}{\beta\theta}} \left(\frac{\beta}{\sigma-1} - 1\right)^{-\frac{1}{\beta}} \gamma_X^{-\frac{1}{\theta}}$.

Equilibrium Productivity

$$g(\varphi) = \begin{cases} \dfrac{f\left(\kappa_D^{\frac{\theta}{\beta}}\gamma_D^{\frac{\sigma-1}{\beta}}\left(\frac{\beta}{\sigma-1}-1\right)^{-\frac{\theta}{\beta}}\varphi^{-\theta}\right)}{F(\gamma_D)}\cdot\Bigg[& \\ \kappa_D^{\frac{\theta}{\beta}}\gamma_D^{\frac{\sigma-1}{\beta}}\left(\dfrac{\beta}{\sigma-1}-1\right)^{-\frac{\theta}{\beta}}\Bigg]\theta\varphi^{-\theta-1} & \text{if } \varphi\in[\varphi_D,\varphi_{DX}) \\ 0 & \text{if } \varphi\in[\varphi_{DX},\varphi_X) \\ \dfrac{f\left(\left(1+\tau^{1-\sigma}\right)\kappa_D^{\frac{\theta}{\beta}}\gamma_D^{\frac{\sigma-1}{\beta}}\left(\frac{\beta}{\sigma-1}-1\right)^{-\frac{\theta}{\beta}}\varphi^{-\theta}\right)}{F(\gamma_D)}\cdot\Bigg[& \text{if } \varphi\in[\varphi_X,\infty) \\ \left(1+\tau^{1-\sigma}\right)\kappa_D^{\frac{\theta}{\beta}}\gamma_D^{\frac{\sigma-1}{\beta}}\left(\dfrac{\beta}{\sigma-1}-1\right)^{-\frac{\theta}{\beta}}\Bigg]\theta\varphi^{-\theta-1} & \end{cases}$$

Equilibrium Productivity

Proposition 1 perfectly holds here.

τ affects the productivity of large firms **directly**.

The country size L does not affect productivity.

General Power Function (GPF) Class

Definition

Definition 3. (Mrazova, Neary and Parenti [2017]) The distribution of φ is of GPF class if its c.d.f. can be expressed as $H\left(\theta_0 + \theta_1 \varphi^{\theta_2}\right)$, where θ_0, θ_1 and θ_2 are parameters, and $H(\cdot)$ is a monotonic function.

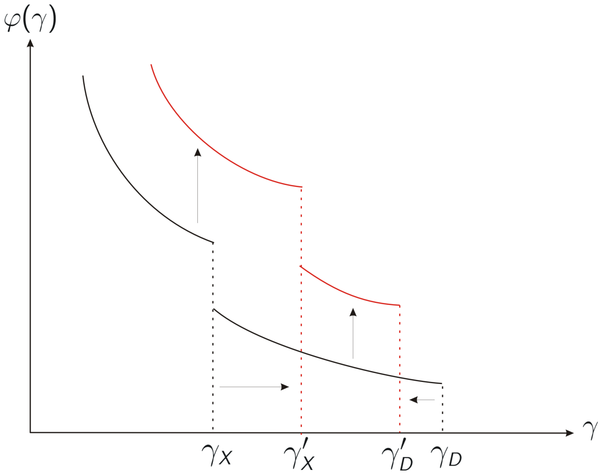
Corollary

Corollary 2. *Let $G(\cdot)$ denotes the c.d.f. of productivity. The productivity distribution belongs to the General Power Function (GPF) with a Pareto tail, where*

$$G(\varphi) = \begin{cases} 1 - F\left(\kappa_D^{\frac{\theta}{\beta}} \gamma_D^{\frac{\sigma-1}{\beta}} \left(\frac{\beta}{\sigma-1} - 1\right)^{-\frac{\theta}{\beta}} \varphi^{-\theta}\right) \frac{1}{F(\gamma_D)} & \text{if } \varphi < \varphi x \\ 1 - F\left(\left(1 + \tau^{1-\sigma}\right) \kappa_D^{\frac{\theta}{\beta}} \gamma_D^{\frac{\sigma-1}{\beta}} \left(\frac{\beta}{\sigma-1} - 1\right)^{-\frac{\theta}{\beta}} \varphi^{-\theta}\right) \frac{1}{F(\gamma_D)} & \text{if } \varphi \geq \varphi x \end{cases}$$

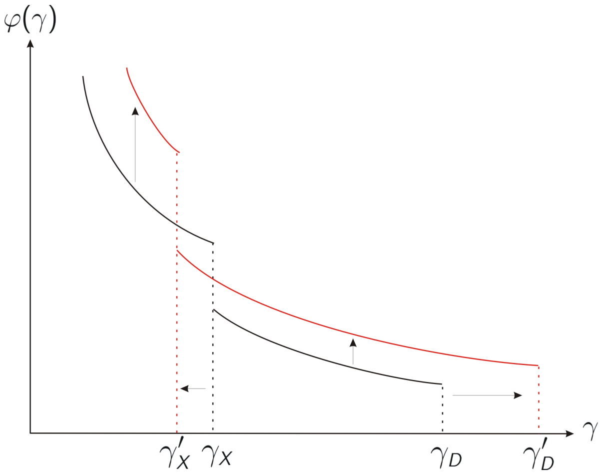
Proposition 2

Figure: The Effect of an Increment of κ_D



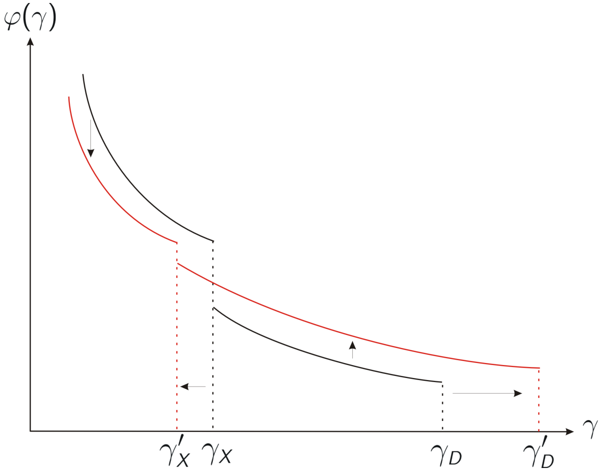
Proposition 2

Figure: The Effect of an Increment of κ_X



Proposition 2

Figure: The Effect of an Increment of τ



Welfare Gains from Trade

Welfare equation of Melitz (2003):

$$d\ln W_0^{ACR} = \frac{d\ln\lambda_0}{1-\sigma-\frac{\eta_{D0}^{ACR}}{\Phi_{D0}}} - \frac{d\ln M_{e0}}{1-\sigma-\frac{\eta_{D0}^{ACR}}{\Phi_{D0}}},$$

where $\frac{\eta_{D0}^{ACR}}{\Phi_{D0}} > 0$.

Welfare equation of productivity model:

$$d\ln W_0 = \frac{d\ln\lambda_0 - d\ln M_{e0} - \frac{\widetilde{\lambda}_{X0}}{\lambda_0}\left(T\xi - \frac{\eta_{X0}}{\Gamma_{X0}}\Xi\right)d\ln\tau}{1-\sigma-(\sigma-1)\frac{\sigma-1}{\theta} - \beta\left(\frac{\widetilde{\lambda}_{D0}}{\lambda_0}\frac{\eta_{D0}}{\Gamma_{D0}} + \frac{\widetilde{\lambda}_{X0}}{\lambda_0}\frac{\eta_{X0}}{\Gamma_{X0}}\right)},$$

where $T > 0$, $\xi < 0$, $\Xi > 0, \widetilde{\lambda}_{X0} > 0$, $\widetilde{\lambda}_{D0} > 0$, $\widetilde{\lambda}_{X0} + \widetilde{\lambda}_{D0} = \lambda_0$, $\frac{\eta_{D0}}{\Gamma_{D0}} > 0$, $\frac{\eta_{X0}}{\Gamma_{X0}} > 0$.

Welfare Gains from Trade

Melitz model has the extensive margin $\eta_{D0}^{ACR}/\Phi_{D0}$ only.

We have:

1. Extensive margin: $\frac{\widetilde{\lambda}_{D0}}{\lambda_0}\frac{\eta_{D0}}{\Gamma_{D0}}$, $\frac{\widetilde{\lambda}_{X0}}{\lambda_0}\frac{\eta_{X0}}{\Gamma_{X0}}$, and $\frac{\widetilde{\lambda}_{X0}}{\lambda_D}\frac{\eta_{X0}}{\Gamma_{X0}}\Xi d\ln\tau$.
   Key: $d\ln\gamma_{X0} = \beta d\ln P_0 - \Xi d\ln\tau$.
2. Intensive margin: $\frac{\widetilde{\lambda}_{X0}}{\lambda_D}T\xi d\ln\tau$ and $(\sigma-1)\,\frac{\sigma-1}{\theta}$
   The direct effect of τ, and the substitution effect.

Welfare Gains from Trade

Benchmark: $g(\varphi) = \theta\varphi^{-\theta-1}$ v.s. $f(\gamma) = \gamma_H^{-1}$.

Melitz:

$$d \ln W_0^{ACR} = \frac{d \ln \lambda_0}{-\theta},$$

$$\varepsilon_{0x}^{ACR} = \varepsilon^{ACR} = -\theta \ \forall x.$$

Our model:

$$d \ln W_0 = -\frac{d \ln \lambda_0}{\beta} + \frac{\widetilde{\lambda}_{X0}}{\lambda_0} \frac{T\xi - \frac{\theta-\sigma+1}{\theta}\Xi}{\beta} d \ln \tau,$$

$$\varepsilon_{0x} = \varepsilon = 1 - \sigma + \xi - \frac{\widetilde{\lambda}_{X0}}{\lambda_0} T\xi - \frac{\theta - \sigma + 1}{\theta}\left(1 - \frac{\widetilde{\lambda}_{X0}}{\lambda_0}\right)\Xi \ \forall x.$$

Welfare Gains from Trade

Within model comparision: firm selection v.s. no firm selection.

(R3) is not important if no firm selection

$$d \ln W_0^{NoSelection} = \frac{d \ln \lambda_0}{1 - \sigma - (\sigma - 1)\frac{\sigma - 1}{\theta}} - \frac{\xi d \ln \tau}{1 - \sigma - (\sigma - 1)\frac{\sigma - 1}{\theta}},$$

$$\varepsilon^{NoSelection} = 1 - \sigma.$$

Similar channel but different magnitude.

Welfare Gains from Trade

Our model v.s. Melitz:

1. The variable trade cost affects the welfare directly: more sensitive to trade.
2. Less welfare elasticity: less sensitive to trade.

Our model with and without selection:

1. Higher welfare elasticity without selection.
2. The effect from $d \ln \tau$ differs.

Conclusion

We obtain the following results under a general setting.

1. Microfundation for power law in productivity and firm size.
2. Intensive margin of productivity matters a lot!
3. Provides empirical insights on the new channel of gains from trade.