Collaborate or Consolidate: Assessing the Competitive Effects of Production Joint Ventures

Nicolas Aguelakakis * Aleksandr Yankelevich †

March 22, 2016

Supplemental Online Appendix

Before we set out to prove the various results and propositions laid out in our manuscript, we restate some of our earlier assumptions using a more general formulation. Three firms indexed 1, 2, and 3 produce imperfectly substitutable goods. Firms are vertically integrated and consist of a separate upstream and downstream division. Each upstream division can produce a unit of an intermediate good at the same constant marginal cost c and with no constraints on capacity. Downstream divisions require one unit of the intermediate good as an input for each unit of output that they produce. Downstream divisions have no other input requirements. Let w_i denote the input price charged by each upstream division to its downstream division. Let θ_i denote the action of the downstream division and let $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3)$ be the profile of all downstream actions. Downstream actions may represent prices, p_i or quantities, x_i.

On the other side of the market, we have a representative consumer who maximizes $\left\{U(\mathbf{x}) - \mathbf{p}\cdot\mathbf{x} : \mathbf{x} \in \mathbf{R}^3_+\right\}$, where $U(\cdot)$ is a symmetric, $\mathbf{C}^3$ (differentially) strictly concave

*Department of Economics, Washington University in St. Louis, Campus Box 1208, St. Louis, MO 63130. e-mail address: naguelakakis@go.wustl.edu

†Corresponding author. Quello Center, Department of Media and Information, Michigan State University, 404 Wilson Road, East Lansing, MI 48824. e-mail address: alyank@msu.edu.

utility on $\mathbf{R}^3_+$, which is (differentially) strictly increasing in a non-empty, bounded set $X \subset \mathbf{R}^3_+$. The utility maximizing consumer gives rise to an inverse demand function f_i for each good i, which is $\mathbf{C}^2$ on the interior of X and decreasing in all its arguments ($\partial f_i/\partial x_j < 0$ for all j). The system of inverse demands can be inverted to yield direct demand functions $x_i = h_i(\mathbf{p})$ which are $\mathbf{C}^2$ in the interior of the region of price space for which demands are positive (denote the region P). When positive, direct demands are downward sloping ($\partial h_i/\partial p_i < 0$ for all i) and yield positive cross effects ($\partial h_i/\partial p_j > 0$ for $i \neq j$). We assume that own effects are larger than cross effects: that is, for $i \neq j$, $|\partial f_i/\partial x_i| > |\partial f_i/\partial x_j|$ and $|\partial h_i/\partial p_i| > \partial h_i/\partial p_j$.

Firms 1 and 2 either merge or join a symmetric input joint venture (JV). A merger preserves both downstream products, but consolidates all decisions. A JV produces and prices the requisite input to be used by its owners, who evenly split the profits of the collaboration, but continue to compete against each other downstream. It is assumed that the firm outside a JV is aware of the ownership and financial division between the JV partners. Within the JV, the input is presumed homogenous, it is bought from the JV if and only if a firm is a party to the JV, parties to the JV are obligated to procure their input from the collaboration, and buying from or selling to outside parties is ruled out by the collaboration contract. Additionally, the marginal cost c of producing the intermediate good does not change in the event of a merger or JV and there are no fixed costs.

For notational convenience, firm profits are written as π_i whether firms compete in prices or quantities downstream. Henceforth, the arguments of the profit function will be used to denote the appropriate competitive scenario: $\pi_i(\mathbf{p})$ for Bertrand, $\pi_i(\mathbf{x})$ for Cournot, and $\pi_i(\boldsymbol{\theta})$ when an expression might apply to either. Moreover, the arguments will be suppressed wherever they are self-evident. Regardless of whether we analyze the baseline or a scenario with a horizontal agreement, we make the following additional assumptions on firm profits, which should be taken to apply to all $\mathbf{p}$ in the interior of P or all $\mathbf{x}$ in the interior of X as appropriate:

Assumption 1. *Firm profits are concave in downstream actions:* $\partial^2\pi_i/\partial\theta_i^2 < 0$.

Assumption 2. *Downstream, prices are strategic complements and quantities are strategic substitutes. That is, for* $i,\ j = 1,\ 2,\ 3,\ i \neq j$*:*

$$\frac{\partial^2\pi_i}{\partial p_i \partial p_j} > 0 \ , \qquad\qquad \frac{\partial^2\pi_i}{\partial x_i \partial x_j} < 0 \ .$$

Consider the Jacobian matrix of the vector of own partials of firm profits:

$$\mathbf{J}_{\boldsymbol{\theta}} = \begin{pmatrix} \dfrac{\partial^2\pi_1}{\partial\theta_1^2} & \dfrac{\partial^2\pi_1}{\partial\theta_1\partial\theta_2} & \dfrac{\partial^2\pi_1}{\partial\theta_1\partial\theta_3} \\ \dfrac{\partial^2\pi_2}{\partial\theta_2\partial\theta_1} & \dfrac{\partial^2\pi_2}{\partial\theta_2^2} & \dfrac{\partial^2\pi_2}{\partial\theta_2\partial\theta_3} \\ \dfrac{\partial^2\pi_3}{\partial\theta_3\partial\theta_1} & \dfrac{\partial^2\pi_3}{\partial\theta_3\partial\theta_2} & \dfrac{\partial^2\pi_3}{\partial\theta_3^2} \end{pmatrix}$$

Assumption 3. *The following stability relationships hold:*

1. *The determinant of* $\mathbf{J}_{\boldsymbol{\theta}}$*,* $|\mathbf{J}_{\boldsymbol{\theta}}|$*, is negative,*
2. $\dfrac{\partial^2\pi_1}{\partial\theta_1^2}\dfrac{\partial^2\pi_2}{\partial\theta_2^2} > \dfrac{\partial^2\pi_1}{\partial\theta_1\partial\theta_2}\dfrac{\partial^2\pi_2}{\partial\theta_2\partial\theta_1}$.

Observe that the first item in Assumption 3 is necessary for the existence of a locally strictly stable equilibrium while the second item preserves this stability in the absence of firm 3. Assumptions 1 and 3 are necessary and sufficient for $\mathbf{J}_{\boldsymbol{\theta}}$ to be negative definite.

Firms play the following two-stage game: In the first stage, firms choose input prices. In the event that firms 1 and 2 are parties to an input JV, the JV chooses a price w that meets the approval of both owners. In the symmetric context discussed here, this is a price that maximizes each owner's total profit. Because absent capacity constraints, the optimal downstream price of a firm with complete ownership and control over its upstream production facility is invariant to the input price set by that facility, we suppose that a firm that is not party to a JV sets $w = c$. At stage two, after learning the input prices, firms simultaneously set downstream prices. The equilibrium concept is SPNE.

Firm profits

Suppose that firms 1 and 2 form a JV. Given an input price w, if firms compete in prices downstream, the profits of firm $i = 1, 2$ are:

$$\pi_i(\mathbf{p}) = (p_i - w)\, h_i(\mathbf{p}) + \frac{w - c}{2}\left[h_1(\mathbf{p}) + h_2(\mathbf{p})\right] \tag{A1}$$

Firm 3's profit equation is given by $\pi_3(\mathbf{p}) = (p_3 - c)h_3(\mathbf{p})$.

When firms compete in quantities downstream, the profits of firm $i = 1, 2$ are:

$$\pi_i(\mathbf{x}) = \left[f_i(\mathbf{x}) - w\right] x_i + \frac{w - c}{2}(x_1 + x_2) \tag{A2}$$

while firm 3's profit becomes $\pi_3(\mathbf{x}) = \left[f_3(\mathbf{x}) - c\right] x_3$.

Let $\mathbf{g}_{\boldsymbol{\theta}} = (\partial\pi_1/\partial\theta_1,\, \partial\pi_2/\partial\theta_2,\, \partial\pi_3/\partial\theta_3)$. The first-order conditions to firms' profit maximization problems in, respectively, the Bertrand and Cournot competitive scenarios are:

$$\mathbf{g}_{\mathbf{p}}\left(\mathbf{p},\, w\right) = \begin{pmatrix} h_1 + (p_1 - w)\,\partial h_1/\partial p_1 + (w - c)\,(\partial h_1/\partial p_1 + \partial h_2/\partial p_1)/2 \\ h_2 + (p_2 - w)\,\partial h_2/\partial p_2 + (w - c)\,(\partial h_1/\partial p_2 + \partial h_2/\partial p_2)/2 \\ h_3 + (p_3 - c)\partial h_3/\partial p_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{A3}$$

$$\mathbf{g}_{\mathbf{x}}\left(\mathbf{x},\, w\right) = \begin{pmatrix} (\partial f_1/\partial x_1)\, x_1 + f_1 - w + (w - c)/2 \\ (\partial f_2/\partial x_2)\, x_2 + f_2 - w + (w - c)/2 \\ (\partial f_3/\partial x_3)\, x_3 + f_3 - c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{A4}$$

Going forward, we restrict w to an open, bounded set, W_p or W_x in $\mathbf{R}$, such that Assumptions 1, 2, and 3 apply to Bertrand or Cournot competition, respectively. Thus, simultaneous solutions to firm first-order conditions as specified by Expressions (A3) or

(A4) lead to a strictly stable equilibrium in prices or quantities, respectively. For a given $w \in W_\theta$, we denote the equilibrium action of firm i as a function of w, $\theta_i(w)$.

Suppose instead that firms 1 and 2 merge. The merged firm's Bertrand profit equation is $\pi_M(\mathbf{p}) = (p_1 - c)h_1(\mathbf{p}) + (p_2 - c)h_2(\mathbf{p})$ and its Cournot profit equation is $\pi_M(\mathbf{x}) = [f_1(\mathbf{x}) - c]\, x_1 + [f_2(\mathbf{x}) - c]\, x_2$. The profit functions for firm 3 remain the same as in the joint venture scenario.

Let $\mathbf{g}_{\boldsymbol{\theta}}^{\mathbf{M}}$ be the vector of own partials of firm profits in the merger scenario. The first-order conditions in, respectively, the Bertrand and Cournot scenarios become:

$$\mathbf{g}_{\mathbf{p}}^{\mathbf{M}}(\mathbf{p}, w) = \begin{pmatrix} h_1 + (p_1 - c)\partial h_1/\partial p_1 + (p_2 - c)\partial h_2/\partial p_1 \\ (p_1 - c)\partial h_1/\partial p_2 + h_2 + (p_2 - c)\partial h_2/\partial p_2 \\ h_3 + (p_3 - c)\partial h_3/\partial p_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{A5}$$

$$\mathbf{g}_{\mathbf{x}}^{\mathbf{M}}(\mathbf{x}, w) = \begin{pmatrix} (\partial f_1/\partial x_1)\, x_1 + f_1 - c + (\partial f_2/\partial x_1)\, x_2 \\ (\partial f_1/\partial x_2)\, x_1 + (\partial f_2/\partial x_2)\, x_2 + f_2 - c \\ (\partial f_3/\partial x_3)\, x_3 + f_3 - c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{A6}$$

Observe that the $\mathbf{g}_{\boldsymbol{\theta}}^{\mathbf{M}}$ are only artificially functions of w, which in this case we interpret as the input price paid by the downstream divisions of the horizontally merged firm. Assuming that $c \in W_\theta$, Assumptions 1, 2, and 3 apply, such that simultaneous solutions to firm first-order conditions as specified by Expressions (A5) or (A6) lead to a strictly stable equilibrium in prices or quantities, respectively. We denote the equilibrium action with regard to product i (where the merged firm controls products 1 and 2), θ_i^M.

Comparative statics

Lemma 1. *Suppose that firms 1 and 2 collaborate in a symmetric input joint venture. If Assumptions 1, 2, and 3 hold, then:*

1. *Under downstream Bertrand competition, equilibrium prices increase in w.*
2. *Under downstream Cournot competition, the equilibrium quantities of firms 1 and 2 decrease in w and the equilibrium quantity of firm 3 increases in w.*

Proof. Let $\mathbf{p}^*$ and $\mathbf{x}^*$, represent the values of $\mathbf{p}$ and $\mathbf{x}$ such that $\mathbf{g_p}(\mathbf{p}^*, w) = \mathbf{0}$ and $\mathbf{g_x}(\mathbf{x}^*, w) = \mathbf{0}$. Note that $\mathbf{g_p}$ and $\mathbf{g_x}$ map from, respectively, int $P \times W_p$ and int $X \times W_x$ into $\mathbb{R}^3_+$. Additionally, define $D_w\mathbf{g}_{\boldsymbol{\theta}}$ as the column vector of own partials differentiated with respect to w. That is,

$$D_w\mathbf{g}_{\boldsymbol{\theta}} = \begin{pmatrix} \frac{\partial^2 \pi_1}{\partial \theta_1 \partial w} & \frac{\partial^2 \pi_2}{\partial \theta_2 \partial w} & \frac{\partial^2 \pi_3}{\partial \theta_3 \partial w} \end{pmatrix}^{\mathsf{T}}$$

From our assumptions on utility along with Assumption 3, we know that we can apply the implicit function theorem to obtain the derivative of firm actions with respect to w. In particular, $\boldsymbol{\theta}^* = \boldsymbol{\theta}(w)$ and $\boldsymbol{\theta}'(w) = -(\mathbf{J}_{\boldsymbol{\theta}})^{-1} D_w\mathbf{g}_{\boldsymbol{\theta}}$. Observe that $(\mathbf{J}_{\boldsymbol{\theta}})^{-1} = (\mathbf{C}_{\boldsymbol{\theta}})^{\mathsf{T}} / |\mathbf{J}_{\boldsymbol{\theta}}|$ where $\mathbf{C}_{\boldsymbol{\theta}}$ is the following cofactor matrix:

$$\begin{pmatrix} \frac{\partial^2 \pi_2}{\partial \theta_2^2}\frac{\partial^2 \pi_3}{\partial \theta_3^2} - \frac{\partial^2 \pi_2}{\partial \theta_2 \partial \theta_3}\frac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_2} & \frac{\partial^2 \pi_2}{\partial \theta_2 \partial \theta_3}\frac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_1} - \frac{\partial^2 \pi_2}{\partial \theta_2 \partial \theta_1}\frac{\partial^2 \pi_3}{\partial \theta_3^2} & \frac{\partial^2 \pi_2}{\partial \theta_2 \partial \theta_1}\frac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_2} - \frac{\partial^2 \pi_2}{\partial \theta_2^2}\frac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_1} \\ \frac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_3}\frac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_2} - \frac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_2}\frac{\partial^2 \pi_3}{\partial \theta_3^2} & \frac{\partial^2 \pi_1}{\partial \theta_1^2}\frac{\partial^2 \pi_3}{\partial \theta_3^2} - \frac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_3}\frac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_1} & \frac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_2}\frac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_1} - \frac{\partial^2 \pi_1}{\partial \theta_1^2}\frac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_2} \\ \frac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_2}\frac{\partial^2 \pi_2}{\partial \theta_2 \partial \theta_3} - \frac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_3}\frac{\partial^2 \pi_2}{\partial \theta_2^2} & \frac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_3}\frac{\partial^2 \pi_2}{\partial \theta_2 \partial \theta_1} - \frac{\partial^2 \pi_1}{\partial \theta_1^2}\frac{\partial^2 \pi_2}{\partial \theta_2 \partial \theta_3} & \frac{\partial^2 \pi_1}{\partial \theta_1^2}\frac{\partial^2 \pi_2}{\partial \theta_2^2} - \frac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_2}\frac{\partial^2 \pi_2}{\partial \theta_2 \partial \theta_1} \end{pmatrix}$$

Our symmetry assumptions on utility, marginal costs, and the division of JV profits imply that $\theta_1^* = \theta_2^*$ along with the following equilibrium relationships on demand and inverse demand:

$$\frac{\partial h_1}{\partial p_1} = \frac{\partial h_2}{\partial p_2}, \quad \frac{\partial h_1}{\partial p_2} = \frac{\partial h_2}{\partial p_1}, \quad \frac{\partial h_1}{\partial p_3} = \frac{\partial h_2}{\partial p_3}, \quad \frac{\partial h_3}{\partial p_1} = \frac{\partial h_3}{\partial p_2}$$

$$\frac{\partial f_1}{\partial x_1} = \frac{\partial f_2}{\partial x_2}, \quad \frac{\partial f_1}{\partial x_2} = \frac{\partial f_2}{\partial x_1}, \quad \frac{\partial f_1}{\partial x_3} = \frac{\partial f_2}{\partial x_3}, \quad \frac{\partial f_3}{\partial x_1} = \frac{\partial f_3}{\partial x_2}$$

$$\frac{\partial^2 h_1}{\partial p_1^2} = \frac{\partial^2 h_2}{\partial p_2^2}, \quad \frac{\partial^2 h_1}{\partial p_2^2} = \frac{\partial^2 h_2}{\partial p_1^2}, \quad \frac{\partial^2 f_1}{\partial x_1^2} = \frac{\partial^2 f_2}{\partial x_2^2}, \quad \frac{\partial^2 f_1}{\partial x_2^2} = \frac{\partial^2 f_2}{\partial x_1^2}$$

$$\frac{\partial^2 h_1}{\partial p_1 \partial p_2} = \frac{\partial^2 h_2}{\partial p_1 \partial p_2}, \quad \frac{\partial^2 h_1}{\partial p_1 \partial p_3} = \frac{\partial^2 h_2}{\partial p_2 \partial p_3}, \quad \frac{\partial^2 h_1}{\partial p_2 \partial p_3} = \frac{\partial^2 h_2}{\partial p_1 \partial p_3}, \quad \frac{\partial^2 h_3}{\partial p_3 \partial p_1} = \frac{\partial^2 h_3}{\partial p_3 \partial p_2}$$

$$\frac{\partial^2 f_1}{\partial x_1 \partial x_2} = \frac{\partial^2 f_2}{\partial x_1 \partial x_2}, \quad \frac{\partial^2 f_1}{\partial x_1 \partial x_3} = \frac{\partial^2 f_2}{\partial x_2 \partial x_3}, \quad \frac{\partial^2 f_1}{\partial x_2 \partial x_3} = \frac{\partial^2 f_2}{\partial x_1 \partial x_3}, \quad \frac{\partial^2 f_3}{\partial x_3 \partial x_1} = \frac{\partial^2 f_3}{\partial x_3 \partial x_2}$$

Our symmetry assumptions also imply the following profit relationships:

$$\frac{\partial^2 \pi_1}{\partial \theta_1^2} = \frac{\partial^2 \pi_2}{\partial \theta_2^2}, \quad \frac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_2} = \frac{\partial^2 \pi_2}{\partial \theta_2 \partial \theta_1}, \quad \frac{\partial^2 \pi_1}{\partial \theta_1 \partial w} = \frac{\partial^2 \pi_2}{\partial \theta_2 \partial w}$$

$$\frac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_3} = \frac{\partial^2 \pi_2}{\partial \theta_2 \partial \theta_3}, \quad \frac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_1} = \frac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_2}$$

Applying the profit relationships above to $\mathbf{J}_{\boldsymbol{\theta}}$ and $\mathbf{C}_{\boldsymbol{\theta}}$ reduces $|\mathbf{J}_{\boldsymbol{\theta}}|$ to:

$$|\mathbf{J}_{\boldsymbol{\theta}}| = \left[\frac{\partial^2 \pi_3}{\partial \theta_3^2}\left(\frac{\partial^2 \pi_1}{\partial \theta_1^2} + \frac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_2}\right) - 2\frac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_3}\frac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_1}\right]\left(\frac{\partial^2 \pi_1}{\partial \theta_1^2} - \frac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_2}\right) \tag{A7}$$

and $\boldsymbol{\theta}'(w)$ to:

$$\boldsymbol{\theta}'(w) = \begin{pmatrix} \dfrac{\partial^2 \pi_1}{\partial \theta_1 \partial w}\dfrac{\partial^2 \pi_3}{\partial \theta_3^2} \Big/ \left[2\dfrac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_3}\dfrac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_1} - \dfrac{\partial^2 \pi_3}{\partial \theta_3^2}\left(\dfrac{\partial^2 \pi_1}{\partial \theta_1^2} + \dfrac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_2}\right)\right] \\ \dfrac{\partial^2 \pi_1}{\partial \theta_1 \partial w}\dfrac{\partial^2 \pi_3}{\partial \theta_3^2} \Big/ \left[2\dfrac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_3}\dfrac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_1} - \dfrac{\partial^2 \pi_3}{\partial \theta_3^2}\left(\dfrac{\partial^2 \pi_1}{\partial \theta_1^2} + \dfrac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_2}\right)\right] \\ 2\dfrac{\partial^2 \pi_1}{\partial \theta_1 \partial w}\dfrac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_1} \Big/ \left[\dfrac{\partial^2 \pi_3}{\partial \theta_3^2}\left(\dfrac{\partial^2 \pi_1}{\partial \theta_1^2} + \dfrac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_2}\right) - 2\dfrac{\partial^2 \pi_1}{\partial \theta_1 \partial \theta_3}\dfrac{\partial^2 \pi_3}{\partial \theta_3 \partial \theta_1}\right] \end{pmatrix}$$

Bertrand: The expression for $\partial^2 \pi_1 / \partial \theta_1 \partial w$ reduces to:

$$\frac{\partial^2 \pi_1}{\partial p_1 \partial w} = \frac{1}{2}\left(\frac{\partial h_2}{\partial p_1} - \frac{\partial h_1}{\partial p_1}\right),$$

which is positive on P. As a result, from Assumption 1 we know that the numerator in

$p_1'(w) = p_2'(w)$ is negative whereas from Assumption 2 for Bertrand competition (strategic complementarity), we know the numerator in $p_3'(w)$ is positive. Moreover, Assumptions 1 and 2 imply that the rightmost parenthetical expression on the right-hand side of Equation (A7) is negative so that by Assumption 3, the denominator in $p_1'(w) = p_2'(w)$ is negative and the denominator in $p_3'(w)$ is positive.

Cournot: The expression for $\partial^2\pi_1/\partial\theta_1\partial w$ now becomes simply $\partial^2\pi_1/\partial x_1\partial w = -(1/2)$. Therefore, from Assumption 1 and Assumption 2 for Cournot competition (strategic substitutability), we know that all the numerators in $\mathbf{x}'(w)$ are positive. Applying our symmetric profit relationships, we can rewrite the inequality found in the second item of Assumption 3 as:

$$\left(\frac{\partial^2\pi_1}{\partial x_1^2} + \frac{\partial^2\pi_1}{\partial x_1\partial x_2}\right)\left(\frac{\partial^2\pi_1}{\partial x_1^2} - \frac{\partial^2\pi_1}{\partial x_1\partial x_2}\right) > 0 \tag{A8}$$

Assumptions 1 and 2 imply that the leftmost parenthetical expression on the left-hand side of Inequality (A8) is negative, which implies the same for the remaining parenthetical expression in the inequality. Observe that the latter parenthetical expression is the Cournot variant of the rightmost parenthetical expression on the right-hand side of Equation (A7), so that according to the first item of Assumption 3, the denominator in $x_1'(w) = x_2'(w)$ is negative and the denominator in $x_3'(w)$ is positive. □

Main Results

Proposition 1. *Suppose that firms 1 and 2 collaborate in a symmetric input joint venture and suppose that firm 3's action is fixed at θ_3^M. Then, the equilibrium input price, $\bar{w}$, is such that $\theta_i(\bar{w}) = \theta_i^M$ for $i = 1, 2, 3$ and $\pi_1(\boldsymbol{\theta}(\bar{w})) + \pi_2(\boldsymbol{\theta}(\bar{w})) = \pi_M(\boldsymbol{\theta}^{\mathbf{M}})$.*

Proof. We approach the proofs for the Bertrand and Cournot scenarios in turn:

Bertrand: When firm 3's price is constant at p_3^M, firm i's, $i \neq j = 1, 2$, first-order condition becomes:

$$\frac{\mathrm{d}\pi_i(\mathbf{p}(w))}{\mathrm{d}w} = \frac{\mathrm{d}p_i}{\mathrm{d}w}h_i + \frac{1}{2}(h_j - h_i) + (p_i - w)\left(\frac{\partial h_i}{\partial p_i}\frac{\mathrm{d}p_i}{\mathrm{d}w} + \frac{\partial h_i}{\partial p_j}\frac{\mathrm{d}p_j}{\mathrm{d}w}\right) + \frac{w-c}{2}\left[\left(\frac{\partial h_i}{\partial p_i} + \frac{\partial h_j}{\partial p_i}\right)\frac{\mathrm{d}p_i}{\mathrm{d}w} + \left(\frac{\partial h_i}{\partial p_j} + \frac{\partial h_j}{\partial p_j}\right)\frac{\mathrm{d}p_j}{\mathrm{d}w}\right] = 0 \quad \text{(A9)}$$

Symmetry implies that in equilibrium, $h_1 = h_2$, $\mathrm{d}p_1/\mathrm{d}w = \mathrm{d}p_2/\mathrm{d}w$, $\partial h_1/\partial p_1 = \partial h_2/\partial p_2$, and $\partial h_2/\partial p_1 = \partial h_1/\partial p_2$. As a result, Equation (A9) reduces to:

$$h_i + (p_i - c)\left(\frac{\partial h_i}{\partial p_i} + \frac{\partial h_j}{\partial p_i}\right) = 0 \quad \text{(A10)}$$

Referring back to Expression (A5) and noting that symmetry also implies that $p_1^M = p_2^M$ (or alternatively, that $p_1(\bar{w}) = p_2(\bar{w})$), we see that Equation (A10) is equivalent to the first-order condition for product i in the horizontal merger scenario. Because firm 3's price is p_3^M by assumption, it follows that $p_i(\bar{w}) = p_i^M$ for $i = 1, 2$ as well. Furthermore, because $p_i(\bar{w}) = p_i^M$ for $i = 1, 2$, p_3^M turns out to be firm 3's best response when the JV sets input price $\bar{w}$, so that we may write $p_3(\bar{w}) = p_3^M$. Consequently, $\pi_1(\mathbf{p}(\bar{w})) + \pi_2(\mathbf{p}(\bar{w})) = (p_1 - c)h_1(\mathbf{p}(\bar{w})) + (p_2 - c)h_2(\mathbf{p}(\bar{w})) = \pi_M(\mathbf{p}^{\mathbf{M}})$.

Cournot: The Cournot proof is analogous to its Bertrand counterpart. That is, when firm 3's quantity is constant at x_3^M, firm i's, $i \neq j = 1, 2$, first-order condition becomes:

$$\frac{\mathrm{d}\pi_i(\mathbf{x}(w))}{\mathrm{d}w} = \left(\frac{\partial f_i}{\partial x_i}\frac{\mathrm{d}x_i}{\mathrm{d}w} + \frac{\partial f_i}{\partial x_j}\frac{\mathrm{d}x_j}{\mathrm{d}w}\right)x_i + (f_i - w)\frac{\mathrm{d}x_i}{\mathrm{d}w} + \frac{1}{2}(x_j - x_i) + \frac{w-c}{2}\left(\frac{\mathrm{d}x_i}{\mathrm{d}w} + \frac{\mathrm{d}x_j}{\mathrm{d}w}\right) = 0 \quad \text{(A11)}$$

Symmetry implies that in equilibrium, $x_1(\bar{w}) = x_2(\bar{w})$ and $\mathrm{d}x_1/\mathrm{d}w = \mathrm{d}x_2/\mathrm{d}w$. As a result, Equation (A11) reduces to:

$$f_i - c + x_i\left(\frac{\partial f_i}{\partial x_i} + \frac{\partial f_i}{\partial x_j}\right) = 0 \quad \text{(A12)}$$

Referring back to Expression (A6) and noting that symmetry also implies that $\partial f_2/\partial x_1 = \partial f_1/\partial x_2$ and $x_1^M = x_2^M$, we see that Equation (A12) is equivalent to the first-order condition for product i in the horizontal merger scenario. Because firm 3's quantity is x_3^M by assumption, it follows that $x_i(\bar{w}) = x_i^M$ for $i = 1, 2$ as well. Furthermore, because $x_i(\bar{w}) = x_i^M$ for $i = 1, 2$, x_3^M turns out to be firm 3's best response when the JV sets input price $\bar{w}$, so that we may write $x_3(\bar{w}) = x_3^M$. Consequently, $\pi_1(\mathbf{x}(\bar{w})) + \pi_2(\mathbf{x}(\bar{w})) = [f_1(\mathbf{x}(\bar{w})) - c]x_1 + [f_2(\mathbf{x}(\bar{w})) - c]x_2 = \pi_M(\mathbf{x^M})$. □

Proposition 2 (**Bertrand**). *Suppose that firms 1 and 2 collaborate in a symmetric input joint venture and firms compete in prices downstream. In equilibrium,* $w^* > \bar{w}$ *and* $p_i(w^*) > p_i^M$, $i = 1, 2, 3$. *Additionally,* $\pi_1(\mathbf{p}(w^*)) + \pi_2(\mathbf{p}(w^*)) > \pi_M(\mathbf{p^M})$ *and* $\pi_3(\mathbf{p}(w^*)) > \pi_3(\mathbf{p^M})$.

Proof. The change in firm i's, $i \neq j = 1, 2$, profit with respect to w can be written:

$$\begin{aligned}\frac{\mathrm{d}\pi_i(\mathbf{p}(w))}{\mathrm{d}w} &= \frac{\mathrm{d}p_i}{\mathrm{d}w}h_i + \frac{1}{2}(h_j - h_i) + (p_i - w)\left(\frac{\partial h_i}{\partial p_i}\frac{\mathrm{d}p_i}{\mathrm{d}w} + \frac{\partial h_i}{\partial p_j}\frac{\mathrm{d}p_j}{\mathrm{d}w}\right) \\ &+ \frac{w-c}{2}\left[\left(\frac{\partial h_i}{\partial p_i} + \frac{\partial h_j}{\partial p_i}\right)\frac{\mathrm{d}p_i}{\mathrm{d}w} + \left(\frac{\partial h_i}{\partial p_j} + \frac{\partial h_j}{\partial p_j}\right)\frac{\mathrm{d}p_j}{\mathrm{d}w}\right] \\ &+ (p_i - w)\frac{\partial h_i}{\partial p_3}\frac{\mathrm{d}p_3}{\mathrm{d}w} + \frac{w-c}{2}\left(\frac{\partial h_i}{\partial p_3} + \frac{\partial h_j}{\partial p_3}\right)\frac{\mathrm{d}p_3}{\mathrm{d}w}\end{aligned} \tag{A13}$$

Symmetry implies that in equilibrium, $h_1 = h_2$, $\mathrm{d}p_1/\mathrm{d}w = \mathrm{d}p_2/\mathrm{d}w$, $\partial h_1/\partial p_1 = \partial h_2/\partial p_2$, $\partial h_2/\partial p_1 = \partial h_1/\partial p_2$, and $\partial h_1/\partial p_3 = \partial h_2/\partial p_3$. As a result, Equation (A13) reduces to:

$$\frac{\mathrm{d}\pi_i(\mathbf{p}(w))}{\mathrm{d}w} = \left[h_i + (p_i - c)\left(\frac{\partial h_i}{\partial p_i} + \frac{\partial h_j}{\partial p_i}\right)\right]\frac{\mathrm{d}p_i}{\mathrm{d}w} + (p_i - c)\frac{\partial h_i}{\partial p_3}\frac{\mathrm{d}p_3}{\mathrm{d}w} \tag{A14}$$

Substituting $\bar{w}$ into Equation (A14) and applying Proposition 1 yields:

$$\left.\frac{\mathrm{d}\pi_i(\mathbf{p}(w))}{\mathrm{d}w}\right|_{\bar{w}} = (p_i - c)\left.\frac{\partial h_i}{\partial p_3}\frac{\mathrm{d}p_3}{\mathrm{d}w}\right|_{\bar{w}} > 0 \tag{A15}$$

where the inequality follows by our assumption that products are gross substitutes and from the first item in Lemma 1. The inequality in Expression (A15) tells us that $\bar{w}$ does not lead to an optimum in the complete game so that by definition, $\pi_i(\mathbf{p}(w^*)) > \pi_i(\mathbf{p}(\bar{w}))$ for $i = 1$, 2 and by Proposition 1, $\pi_1(\mathbf{p}(w^*)) + \pi_2(\mathbf{p}(w^*)) > \pi_M(\mathbf{p^M})$.

Now suppose that contrary to the statement of the Proposition, $w^* < \bar{w}$. This leads to the following contradiction:

$$\begin{aligned}
\pi_1(\mathbf{p}(w^*)) + \pi_2(\mathbf{p}(w^*)) &= \pi_M(\mathbf{p}(w^*)) \\
&< \pi_M(p_1(w^*), p_2(w^*), p_3(\bar{w})) \\
&< \pi_M(\mathbf{p}(\bar{w})) \\
&= \pi_M(\mathbf{p^M}) < \pi_1(\mathbf{p}(w^*)) + \pi_2(\mathbf{p}(w^*))
\end{aligned}$$

The initial equality follows from symmetry. The first inequality follows from Lemma 1 (whereby $w^* < \bar{w}$ implies that $p_3(w^*) < p_3(\bar{w})$) together with gross substitutability. The remaining relations follow from Proposition 1. We have thus proven that $w^* > \bar{w}$. From Lemma 1 it follows that $p_i(w^*) > p_i^M$, $i = 1$, 2, 3.

It remains to show that $\pi_3(\mathbf{p}(w^*)) > \pi_3(\mathbf{p^M})$. The change in firm 3's profit with respect to w is given by:

$$\begin{aligned}
\frac{\mathrm{d}\pi_3(\mathbf{p}(w))}{\mathrm{d}w} &= \frac{\mathrm{d}p_3}{\mathrm{d}w} h_3 + (p_3 - c)\left(\frac{\partial h_3}{\partial p_1}\frac{\mathrm{d}p_1}{\mathrm{d}w} + \frac{\partial h_3}{\partial p_2}\frac{\mathrm{d}p_2}{\mathrm{d}w} + \frac{\partial h_3}{\partial p_3}\frac{\mathrm{d}p_3}{\mathrm{d}w}\right) \\
&= (p_3 - c)\left(\frac{\partial h_3}{\partial p_1}\frac{\mathrm{d}p_1}{\mathrm{d}w} + \frac{\partial h_3}{\partial p_2}\frac{\mathrm{d}p_2}{\mathrm{d}w}\right) > 0
\end{aligned}$$

The second equality follows from firm 3's second stage first-order condition (see Expression (A3)) and the inequality follows from Lemma 1 together with gross substitutability. The proof follows from Proposition 1 because $w^* > \bar{w}$. □

Proposition 2 (Cournot). *Suppose that firms 1 and 2 collaborate in a symmetric input joint venture and firms compete in quantities downstream. In equilibrium,* $\bar{w} > w^*$ *and* $x_i(w^*) > x_i^M$, $i = 1, 2$ *whereas* $x_3^M > x_3(w^*)$. *Additionally,* $\pi_1(\mathbf{x}(w^*)) + \pi_2(\mathbf{x}(w^*)) > \pi_M(\mathbf{x^M})$ *whereas* $\pi_3(\mathbf{x^M}) > \pi_3(\mathbf{x}(w^*))$.

Proof. The change in firm i's, $i \neq j = 1$, 2, profit with respect to w can be written:

$$\frac{\mathrm{d}\pi_i(\mathbf{x}(w))}{\mathrm{d}w} = \left(\frac{\partial f_i}{\partial x_i}\frac{\mathrm{d}x_i}{\mathrm{d}w} + \frac{\partial f_i}{\partial x_j}\frac{\mathrm{d}x_j}{\mathrm{d}w} + \frac{\partial f_i}{\partial x_3}\frac{\mathrm{d}x_3}{\mathrm{d}w}\right) x_i + (f_i - w)\frac{\mathrm{d}x_i}{\mathrm{d}w} + \frac{1}{2}(x_j - x_i) + \frac{w-c}{2}\left(\frac{\mathrm{d}x_i}{\mathrm{d}w} + \frac{\mathrm{d}x_j}{\mathrm{d}w}\right) \tag{A16}$$

Symmetry implies that in equilibrium, $x_1(w^*) = x_2(w^*)$ and $\mathrm{d}x_1/\mathrm{d}w = \mathrm{d}x_2/\mathrm{d}w$. As a result, Equation (A16) reduces to:

$$\frac{\mathrm{d}\pi_i(\mathbf{x}(w))}{\mathrm{d}w} = \left[f_i - c + x_i\left(\frac{\partial f_i}{\partial x_i} + \frac{\partial f_i}{\partial x_j}\right)\right]\frac{\mathrm{d}x_i}{\mathrm{d}w} + x_i\frac{\partial f_i}{\partial x_3}\frac{\mathrm{d}x_3}{\mathrm{d}w} \tag{A17}$$

Substituting $\bar{w}$ into Equation (A17) and applying Proposition 1 yields:

$$\left.\frac{\mathrm{d}\pi_i(\mathbf{x}(w))}{\mathrm{d}w}\right|_{\bar{w}} = x_i \left.\frac{\partial f_i}{\partial x_3}\frac{\mathrm{d}x_3}{\mathrm{d}w}\right|_{\bar{w}} < 0 \tag{A18}$$

where the inequality follows by our assumption that products are substitutes and from the second item in Lemma 1. The inequality in Expression (A18) tells us that $\bar{w}$ does not lead to an optimum in the complete game so that by definition, $\pi_i(\mathbf{x}(w^*)) > \pi_i(\mathbf{x}(\bar{w}))$ for $i = 1, 2$ and by Proposition 1, $\pi_1(\mathbf{x}(w^*)) + \pi_2(\mathbf{x}(w^*)) > \pi_M(\mathbf{x^M})$.

Now suppose that contrary to the statement of the Proposition, $\bar{w} < w^*$. This leads to the following contradiction:

$$\begin{aligned}
\pi_1(\mathbf{x}(w^*)) + \pi_2(\mathbf{x}(w^*)) &= \pi_M(\mathbf{x}(w^*)) \\
&< \pi_M(x_1(w^*), x_2(w^*), x_3(\bar{w})) \\
&< \pi_M(\mathbf{x}(\bar{w})) \\
&= \pi_M(\mathbf{x^M}) < \pi_1(\mathbf{x}(w^*)) + \pi_2(\mathbf{x}(w^*))
\end{aligned}$$

The initial equality follows from symmetry. The first inequality follows from Lemma 1 (whereby $\bar{w} < w^*$ implies that $x_3(\bar{w}) < x_3(w^*)$) together with substitutability. The remaining relations follow from Proposition 1. We have thus proven that $\bar{w} > w^*$. From Lemma 1 it follows that $x_i(w^*) > x_i^M$, $i = 1, 2$ and $x_3^M > x_3(w^*)$.

It remains to show that $\pi_3(\mathbf{x^M}) > \pi_3(\mathbf{x}(w^*))$. The change in firm 3's profit with respect to w is given by:

$$\frac{\mathrm{d}\pi_3(\mathbf{x}(w))}{\mathrm{d}w} = x_3\left(\frac{\partial f_3}{\partial x_1}\frac{\mathrm{d}x_1}{\mathrm{d}w} + \frac{\partial f_3}{\partial x_2}\frac{\mathrm{d}x_2}{\mathrm{d}w} + \frac{\partial f_3}{\partial x_3}\frac{\mathrm{d}x_3}{\mathrm{d}w}\right) + (f_3 - c)\frac{\mathrm{d}x_3}{\mathrm{d}w}$$

$$= x_3\left(\frac{\partial f_3}{\partial x_1}\frac{\mathrm{d}x_1}{\mathrm{d}w} + \frac{\partial f_3}{\partial x_2}\frac{\mathrm{d}x_2}{\mathrm{d}w}\right) > 0$$

The second equality follows from firm 3's second stage first-order condition (see Expression (A4)) and the inequality follows from Lemma 1 together with substitutability. The proof follows from Proposition 1 because $\bar{w} > w^*$. □

Proposition 3. *Suppose that firms 1 and 2 collaborate in a symmetric input joint venture. Then, in equilibrium, firm downstream actions and profits are the same as those that would result had firm 1 and 2 merged and obtained a Stackelberg leadership advantage with respect to firm 3.*

Proof. We approach the proofs for the Bertrand and Cournot scenarios in turn:

Bertrand: Suppose that a merged firm consisting of firms 1 and 2 becomes a Stackelberg leader. Working backwards, firm 3 maximizes its profit as a function of p_1 and p_2 by solving its first-order condition given in Expression (A5) to yield equilibrium price $p_3(p_1, p_2)$. Taking $p_3(p_1, p_2)$ into account, the merged firm's profit equation becomes $\pi_M(p_1, p_2, p_3(p_1, p_2)) = (p_1 - c)h_1(p_1, p_2, p_3(p_1, p_2)) + (p_2 - c)h_2(p_1, p_2, p_3(p_1, p_2))$. Its first-order condition for product $i \neq j \in \{1, 2\}$ becomes:

$$h_i + (p_i - c)\left(\frac{\partial h_i}{\partial p_i} + \frac{\partial h_i}{\partial p_3}\frac{\partial p_3}{\partial p_i}\right) + (p_j - c)\left(\frac{\partial h_j}{\partial p_i} + \frac{\partial h_j}{\partial p_3}\frac{\partial p_3}{\partial p_i}\right) = 0 \tag{A19}$$

Now suppose instead that firms 1 and 2 join a symmetric JV. Recall that the derivative of JV partner profits with respect to w can be written according to Equation (A14) in the proof of Proposition 2 (Bertrand). By using the envelope theorem, the right most derivative in Equation (A14) can be written:

$$\frac{\mathrm{d}p_3}{\mathrm{d}w} = \frac{\partial p_3}{\partial p_1}\frac{\mathrm{d}p_1}{\mathrm{d}w} + \frac{\partial p_3}{\partial p_2}\frac{\mathrm{d}p_2}{\mathrm{d}w} \tag{A20}$$

Substituting back into Equation (A14) and relying on symmetry, the first-order condition for JV firm $i \neq j \in \{1, 2\}$ becomes:

$$\frac{\mathrm{d}p_1}{\mathrm{d}w}\left[h_i + (p_i - c)\left(\frac{\partial h_i}{\partial p_i} + \frac{\partial h_i}{\partial p_3}\frac{\partial p_3}{\partial p_i}\right) + (p_i - c)\left(\frac{\partial h_i}{\partial p_j} + \frac{\partial h_i}{\partial p_3}\frac{\partial p_3}{\partial p_j}\right)\right] = 0 \qquad \text{(A21)}$$

Symmetry implies that the square bracket term in Equation (A21) is equal to the left-hand side in Equation (A19), completing the proof.

Cournot: The Cournot proof proceeds its Bertrand counterpart, but instead relying on Expression (A6) in place of Expression (A5) and Equation (A17) in the proof of Proposition 2 (Cournot) in place of Equation (A14). □

Linear Example

Consider our general model with the following quadratic utility specification:

$$U(\mathbf{x}) = \alpha(x_1 + x_2 + x_3) - \kappa\left(x_1^2 + x_2^2 + x_3^2\right)/2 - \beta(x_1x_2 + x_1x_3 + x_3x_2) \qquad \text{(A22)}$$

where α, κ, and β are positive and $\kappa > \beta$. This utility function gives rise to a linear demand structure with the inverse demand for product i given by:

$$p_i = \alpha - \kappa x_i - \beta\sum_{j\neq i} x_j \qquad \text{(A23)}$$

in the region of X where prices are positive. Solving the system of 3 inverse demand equations for $i = 1, 2, 3$ yields the direct demand for product i in the region of P over which quantities are positive:

$$x_i = a - kp_i + b\sum_{j\neq i} p_j \qquad \text{(A24)}$$

where we write $\alpha = a/(k-2b)$, $\kappa = (k-b)/\left[(k+b)(k-2b)\right]$, and $\beta = b/\left[(k+b)(k-2b)\right]$, and where a, k, and b are positive and $k > 2b$. In addition to our utility assumptions, without loss of generality, suppose that the marginal cost c is zero.

Working backwards, given an input price $w_p \in W_p$ or $w_x \in W_x$, we can solve firms' first-order conditions under Bertrand (Expression (A3)) or Cournot (Expression (A4)) competition, respectively to yield firms' conditional equilibrium actions. Specifically, for $i = 1, 2$ these are

$$\begin{aligned} p_i(w_p) &= \frac{a}{2(k-b)} + \frac{k(k+b)w_p}{2(2k+b)(k-b)} \ , \\ p_3(w_p) &= \frac{a}{2(k-b)} + \frac{b(k+b)w_p}{2(2k+b)(k-b)} \end{aligned} \tag{A25}$$

under Bertrand competition and

$$\begin{aligned} x_i(w_x) &= \frac{\alpha}{2(\kappa+\beta)} - \frac{\kappa w_x}{2(2\kappa-\beta)(\kappa+\beta)} \ , \\ x_3(w_x) &= \frac{\alpha}{2(\kappa+\beta)} + \frac{\beta w_x}{2(2\kappa-\beta)(\kappa+\beta)} \end{aligned} \tag{A26}$$

under Cournot competition. Observe that because $k > 2b > 0$ and $\kappa > \beta > 0$, $p_i(w_p) > p_3(w_p)$ for any $w_p > 0$ and $x_i(w_x) < x_3(w_x)$ for any $w_x > 0$.

We can now substitute $\mathbf{p}(w_p)$ into Equation (A1) and $\mathbf{x}(w_x)$ into Equation (A2) to solve for the equilibrium input prices:

$$w_p^* = \frac{a(2k+b)b}{2(k^2-bk-b^2)k} \ , \qquad w_x^* = \frac{\alpha(\kappa-\beta)(2\kappa-\beta)\beta}{2(\kappa^2+\kappa\beta-\beta^2)} \tag{A27}$$

which are both positive given our assumptions on utility. Substituting w_p^* and w_x^* into Equations (A25) and (A26), respectively, we can obtain the JV equilibrium prices, quantities, and profits under Bertrand and Cournot competition.

Proposition 4. *Suppose that firms 1 and 2 collaborate in a symmetric input joint venture and that firms face linear demand. In equilibrium, the combined profits of firms 1 and 2 are higher than the profits of a horizontal merger between firms 1 and 2. Additionally:*

1. *Under downstream Bertrand competition, the equilibrium profit and quantity of firm 3 and all prices are higher than in the horizontal merger scenario. The quantities of*

products 1 and 2 and total and consumer welfare are lower.

2. *Under downstream Cournot competition, the equilibrium quantities of firms 1 and 2 and total and consumer welfare are higher than in the horizontal merger scenario. All prices, as well as the profit and quantity of firm 3 are lower.*

Proof. Using Table 1, we can compare prices, quantities, and profits for firms $i = 1$, 2 and 3 in the joint venture scenario with the corresponding variables had firms 1 and 2 merged instead when all firms compete in prices downstream. The superscript M_p represents the merger scenario with downstream Bertrand competition. The results regarding prices, quantities, and profits are now easily confirmed by comparing each row.

Table 1: Bertrand Equilibrium: Joint Venture vs. Horizontal Merger

	Joint Venture	Horizontal Merger
Firm i	$p_i(w_p^*) = \frac{a(2k+b)}{4(k^2-kb-b^2)}$	$p_i^{M_p} = \frac{a(2k+b)}{2(2k^2-2kb-b^2)}$
	$x_i(w_p^*) = \frac{a(2k+b)}{4k}$	$x_i^{M_p} = \frac{a(2k+b)(k-b)}{2(2k^2-2kb-b^2)}$
	$\pi_i(\mathbf{p}(w_p^*)) = \frac{a^2(2k+b)^2}{16k(k^2-kb-b^2)}$	$\pi_M(\mathbf{p}^{\mathbf{M}_p}) = \frac{a^2(2k+b)^2(k-b)}{2(2k^2-2kb-b^2)^2}$
Firm 3	$p_3(w_p^*) = \frac{a(2k^2-b^2)}{4k(k^2-kb-b^2)}$	$p_3^{M_p} = \frac{ak}{2k^2-2kb-b^2}$
	$x_3(w_p^*) = \frac{a(2k^2-b^2)}{4(k^2-kb-b^2)}$	$x_3^{M_p} = \frac{ak^2}{2k^2-2kb-b^2}$
	$\pi_3(\mathbf{p}(w_p^*)) = \frac{a^2(2k^2-b^2)^2}{16k(k^2-kb-b^2)^2}$	$\pi_3(\mathbf{p}^{\mathbf{M}_p}) = \frac{a^2k^3}{(2k^2-2kb-b^2)^2}$

To see that consumer and total welfare are lower in the joint venture scenario than in the horizontal merger scenario, we substitute the equilibrium quantities in Table 1 into Equation (A22) and rewrite α, κ, and β in terms of a, k, and b. Total welfare is lower in the joint venture scenario than in the horizontal merger scenario if $U\left(\mathbf{x}(w_p^*)\right) < U\left(\mathbf{x}^{\mathbf{M}_p}\right)$. After some straightforward algebraic manipulation, this inequality reduces to:

$$\frac{-a^2b^2(2k+b)\left(16k^5 - 32k^4b - 20k^3b^2 + 30k^2b^3 + 24kb^4 + 5b^5\right)}{32k\left(2k^2 - 2kb - b^2\right)^2\left(k^2 - bk - b^2\right)^2} < 0 \tag{A28}$$

Similarly, consumer welfare is lower in the joint venture scenario than in the horizontal merger scenario if $U\left(\mathbf{x}(w_p^*)\right) - \mathbf{p}(w_p^*) \cdot \mathbf{x}(w_p^*) < U\left(\mathbf{x}^{\mathbf{M}_p}\right) - \mathbf{p}^{\mathbf{M}_p} \cdot \mathbf{x}^{\mathbf{M}_p}$, which may be rewritten as:

$$\frac{-a^2b^2(2k+b)\left(16k^5 - 16k^4b - 28k^3b^2 + 10k^2b^3 + 16kb^4 + 3b^5\right)}{32k\left(2k^2 - 2kb - b^2\right)^2\left(k^2 - bk - b^2\right)^2} < 0 \tag{A29}$$

Without loss of generality, we may normalize k to 1 in Inequalities (A28) and (A29) to see that under our assumptions (in particular, $b < k/2$), total and consumer welfare decline when firms 1 and 2 form a JV instead of merging horizontally and firms compete in prices downstream.

Table 2 presents the analogous price, quantity, and profit comparison to Table 1 in the event of quantity competition downstream. The superscript M_x represents the merger scenario with downstream Cournot competition. We can now similarly confirm the results regarding prices, quantities, and profits under Cournot competition downstream.

Total welfare is higher in the joint venture scenario than in the horizontal merger scenario if the following inequality holds:

$$\frac{\alpha^2\beta^2(2\kappa-\beta)\left(16\kappa^5 + 16\kappa^4\beta - 28\kappa^3\beta^2 - 10\beta^3\kappa^2 + 16\kappa\beta^4 - 3\beta^5\right)}{32\kappa\left(\kappa^2 + \kappa\beta - \beta^2\right)^2\left(2\kappa^2 + 2\beta\kappa - \beta^2\right)^2} > 0 \tag{A30}$$

Likewise, consumer welfare is higher in the joint venture scenario than in the horizontal merger scenario if:

$$\frac{\alpha^2\beta^2(2\kappa-\beta)\left(16\kappa^5 + 32\kappa^4\beta - 20\kappa^3\beta^2 - 30\beta^3\kappa^2 + 24\kappa\beta^4 - 5\beta^5\right)}{32\kappa\left(\kappa^2 + \kappa\beta - \beta^2\right)^2\left(2\kappa^2 + 2\beta\kappa - \beta^2\right)^2} > 0 \tag{A31}$$

Without loss of generality, we may normalize κ to 1 in Inequalities (A30) and (A31) to see that total and consumer welfare increase when firms 1 and 2 form a JV instead of merging

Table 2: Cournot Equilibrium: Joint Venture vs. Horizontal Merger

	Joint Venture	Horizontal Merger
Firm i	$p_i(w_x^*) = \dfrac{\alpha(2\kappa - \beta)}{4\kappa}$	$p_i^{M_x} = \dfrac{\alpha(2\kappa - \beta)(\kappa + \beta)}{2(2\kappa^2 + 2\kappa\beta - \beta^2)}$
	$x_i(w_x^*) = \dfrac{\alpha(2\kappa - \beta)}{4(\kappa^2 + \kappa\beta - \beta^2)}$	$x_i^{M_x} = \dfrac{\alpha(2\kappa - \beta)}{2(2\kappa^2 + 2\kappa\beta - \beta^2)}$
	$\pi_i(\mathbf{x}(w_x^*)) = \dfrac{\alpha^2(2\kappa - \beta)^2}{16\kappa(\kappa^2 + \kappa\beta - \beta^2)}$	$\pi_M(\mathbf{x}^{\mathbf{M}_x}) = \dfrac{\alpha^2(2\kappa - \beta)^2(\kappa + \beta)}{2(2\kappa^2 + 2\kappa\beta - \beta^2)^2}$
Firm 3	$p_3(w_x^*) = \dfrac{\alpha(2\kappa^2 - \beta^2)}{4(\kappa^2 + \kappa\beta - \beta^2)}$	$p_3^{M_x} = \dfrac{\alpha\kappa^2}{2\kappa^2 + 2\kappa\beta - \beta^2}$
	$x_3(w_x^*) = \dfrac{\alpha(2\kappa^2 - \beta^2)}{4\kappa(\kappa^2 + \kappa\beta - \beta^2)}$	$x_3^{M_x} = \dfrac{\alpha\kappa}{2\kappa^2 + 2\kappa\beta - \beta^2}$
	$\pi_3(\mathbf{x}(w_x^*)) = \dfrac{\alpha^2(2\kappa^2 - \beta^2)^2}{16\kappa(\kappa^2 + \kappa\beta - \beta^2)^2}$	$\pi_3(\mathbf{x}^{\mathbf{M}_x}) = \dfrac{\alpha^2\kappa^3}{(2\kappa^2 + 2\kappa\beta - \beta^2)^2}$

horizontally and firms compete in quantities downstream. □

Extensions (Why firms merge)

Imperfect Information. Suppose that firm 3 does not learn the input price set by the JV prior to downstream competition taking place. Let $\gamma \in W_\theta$ represent firm 3's belief regarding w and let $\theta_i(\theta_j,\, \theta_3;\, w,\, \gamma)$ represent the best response of firm $i \neq j = 1, 2$ given any $w \in W_\theta$ and γ. Let $\tilde{\theta}(w, \gamma)$ solve $\tilde{\theta}(w, \gamma) = \theta_i(\tilde{\theta}(w, \gamma),\, \theta_3(\gamma);\, w,\, \gamma)$.[1]

Proposition 5. *Suppose that firms 1 and 2 collaborate in a symmetric input joint venture in the two stage imperfect information game. Then the assessment consisting of the joint venture playing $\bar{w}$, followed by firms 1 and 2 playing $\tilde{\theta}(w, \gamma)$ for any w and firm 3 playing $\theta_3(\bar{w})$ accompanied by the belief that $\bar{w}$ was played with probability one, constitutes*

[1] Existence of $\tilde{\theta}(w, \gamma)$ follows from symmetry and Assumptions 1 and 3. Moreover, firm 3's action $\theta_3(\gamma)$ is the same that would prevail in a subgame of the baseline game following $w = \gamma$.

a sequential equilibrium. When demand is linear, this sequential equilibrium is unique.

Proof. We first show that the assessment consisting of the JV playing $\bar{w}$, followed by firms 1 and 2 playing $\tilde{\theta}(w, \gamma)$ for any $w \in W_\theta$ and firm 3 playing $\theta_3(\bar{w})$ accompanied by the belief that $\gamma = \bar{w}$ was played with probability one, constitutes a sequential equilibrium of the two stage joint venture game of imperfect information. To simplify the exposition, let us proceed with the extensive form transformation of the second simultaneous move stage in which firm 1's move is followed by that of firm 2, which is followed by that of firm 3 and in which subsequent movers are not made aware of the previous history of the stage. This extensive form specification requires us to additionally specify beliefs about prior downstream actions of firms 2 and 3. Let us suppose that in equilibrium, firm 2 believes that firm 1 plays $\tilde{\theta}(w, \gamma)$ with probability one contingent on w having been played in stage one and that firm 3 believes that firm $i = 1, 2$ plays $\theta_i(\bar{w})$ with probability one.

The sequential rationality of the assessment above follows because given γ, firms 1 and 2 select the optimal downstream action $\tilde{\theta}(w, \gamma)$ at every information set (choice of w) and from the proof of Proposition 1, where we showed that the JV optimizes by choosing $\bar{w}$ when firm 3's action is fixed at $\theta_3(\bar{w})$ and that in turn, $\theta_3(\bar{w})$ is a best response to $\tilde{\theta}(\bar{w}, \bar{w}) = \theta_i(\bar{w}) = \theta_j(\bar{w})$, $i, j = 1, 2$, where $\tilde{\theta}(\bar{w}, \bar{w}) = \theta_i(\bar{w})$ follows because firm 3 plays $\theta_3(\bar{w})$.

Next, let Θ denote the set of downstream actions (P or X as appropriate). In order to show that the assessment is consistent, we first define the following density functions, each of which is positive on the interior of their supports: $\varphi^\epsilon_{JV} : W_\theta \to [0, 1]$, $\varphi^\epsilon_3 : \Theta \to [0, 1]$ and conditional density $\varphi^\epsilon_i : \Theta \times W_\theta \to [0, 1]$, $i = 1, 2$, which is conditional on $w \in W_\theta$, and where the superscript ϵ represents a positive integer. Further, suppose that $\lim_{\epsilon\to\infty} \varphi^\epsilon_{JV}(\bar{w}) = 1$, $\lim_{\epsilon\to\infty} \varphi^\epsilon_3(\theta_3(\bar{w})) = 1$, and $\lim_{\epsilon\to\infty} \varphi^\epsilon_i(\tilde{\theta}(w, \gamma)|w) = 1$.

To show consistency, we may now define a sequence of assessments consisting of completely mixed strategies σ^ϵ and Bayes' rule derived beliefs μ^ϵ which converge to the assessment above. For each ϵ, define the strategy of the JV as $\sigma^\epsilon_{JV}(\varnothing)(w) = \varphi^\epsilon_{JV}(w)$, where the first set of parenthesis on the left-hand side denotes each player's information set. Likewise, define the strategy of firm 1 conditional on w as $\sigma^\epsilon_1(w)(\theta_1) = \varphi^\epsilon_1(\theta_1|w)$, the strategy of firm 2 conditional on w as $\sigma^\epsilon_2(w \times \Theta)(\theta_2) = \varphi^\epsilon_2(\theta_2|w)$, and the strategy of firm 3 as $\sigma^\epsilon_3(W_\theta \times \Theta \times \Theta)(\theta_3) = \varphi^\epsilon_3(\theta_3)$. Proceeding according to the extensive form transformation above, for each ϵ, we may define the beliefs of firm 1 as $\mu^\epsilon_1(w)(w) = \varphi^\epsilon_{JV}(w)$, the beliefs of firm 2 as $\mu^\epsilon_2(w \times \Theta)(w, \theta_1) = \varphi^\epsilon_{JV}(w)\varphi^\epsilon_1(\theta_1|w)$, and the beliefs of firm 3 as $\mu^\epsilon_3(W_\theta \times \Theta \times \Theta)(w, \theta_1, \theta_2) = \varphi^\epsilon_{JV}(w)\varphi^\epsilon_1(\theta_1|w)\varphi^\epsilon_2(\theta_2|w)$.[2] It becomes immediately apparent that the sequence of strategies and beliefs converges to the assessment above and that for each ϵ, beliefs are defined from strategies according to Bayes' rule, such that the assessment is indeed consistent.

Next, suppose that demand is linear and consider downstream competition in prices (again normalizing marginal cost to zero). Let γ_p represent firm 3's belief about w_p. Substituting γ_p into firms' downstream Bertrand profit functions yields $p_i(\gamma_p)$, $i = 1, 2, 3$, where $p_i(\gamma_p)$ is defined by replacing w_p with γ_p in Equation (A25). Next, for $i \neq j = 1, 2$, substitute $h_i(\mathbf{p})$ in Equation (A1) with $a - kp_i + bp_j + bp_3(\gamma_p)$ and solve the system of equations that arises from firm 1 and 2's simultaneous profit maximization problems with respect to downstream prices. Substituting the resulting prices for firms 1 and 2 back into Equation (A1) (as modified in the previous sentence) and maximizing with respect to w_p yields:

$$w_p(\gamma_p) = \frac{b(4ak^2 + b^2\gamma_p k + b^3\gamma_p - ab^2)}{2(k-b)^2(k+b)(2k+b)}$$

Because firm 3's belief must be correct in equilibrium, it must be that $\gamma_p = w_p(\gamma_p)$. This

[2]Note that beliefs for firm 1 are stated redundantly. Moreover, the second set of parenthesis on beliefs refers to all relevant actions. For instance, μ^ϵ_3 specifies firm 3's belief that w, θ_1, and θ_2 will be played.

occurs if and only if

$$\gamma_p = \frac{ab(2k+b)}{(k+b)(2k^2-2bk-b^2)} = \bar{w}_p, \tag{A32}$$

where the second equality is verified by substituting Equation (A32) into Equations (A1) and (A25) and by comparing the resulting profits and prices with the rightmost column in Table 1. Any other belief is inconsistent.[3] The proof for downstream quantity competition follows analogously, but using Equations (A2) and (A26) in place of Equations (A1) and (A25), respectively. □

Cost Reduction. In this subsection we present a simple illustration of the role that cost synergies can play in making a merger at least as profitable as a JV in the Bertrand variant of our model with linear demand. Thus, suppose that as before, there are no synergies following a JV, but that post-merger, the merged firm faces marginal cost $c - t$, where $t > 0$. To economize on notation, as in our earlier linear example, we continue to treat prices as price-cost margins by setting c to zero (so that t can be viewed as increasing the price-cost margin at any price). Given any value of a, k, and b, we can solve for the value of t that equates $2\pi_i(\mathbf{p}(w_p^*))$, $i = 1, 2$ with $\pi_M(\mathbf{p}^{\mathbf{M}_p})$ from Table 1, where $p_i^{M_p}$ is replaced by:

$$p_i^{M_p} = \frac{a(2k+b) - 2tk(k-b)}{2(2k^2-2kb-b^2)} \tag{A33}$$

and $p_3^{M_p}$ is replaced by:

$$p_3^{M_p} = \frac{ak - tb(k-b)}{2k^2-2kb-b^2} \tag{A34}$$

Comparing the price in Equation (A33) with its counterpart in Table 1, it can be observed that the merged firm passes on part of its cost savings to consumers, and thus

[3] We note that it is easy to show that Assumptions 1, 2 and 3 are satisfied and the profit function for firm $i = 1, 2$ is strictly concave in w_p regardless of γ_p. Moreover, γ_p drops out of the second derivative of firm i's profit. Thus, firm 3's belief cannot be a mixture.

continues to price below a JV without synergies. Firm 3 similarly lowers its price compared to that following a merger without synergies. In Figure 1, we set $a = 100$, $k = 1$ and examine the value of t such that $2\pi_i(\mathbf{p}(w_p^*)) = \pi_M(\mathbf{p}^{\mathbf{M}_p})$ for all values of $b \in [0, 0.5)$ (recall that by assumption, $k > 2b$).

Figure 1: Value of t Such That Firms 1 and 2 are Indifferent Between JV and Merger

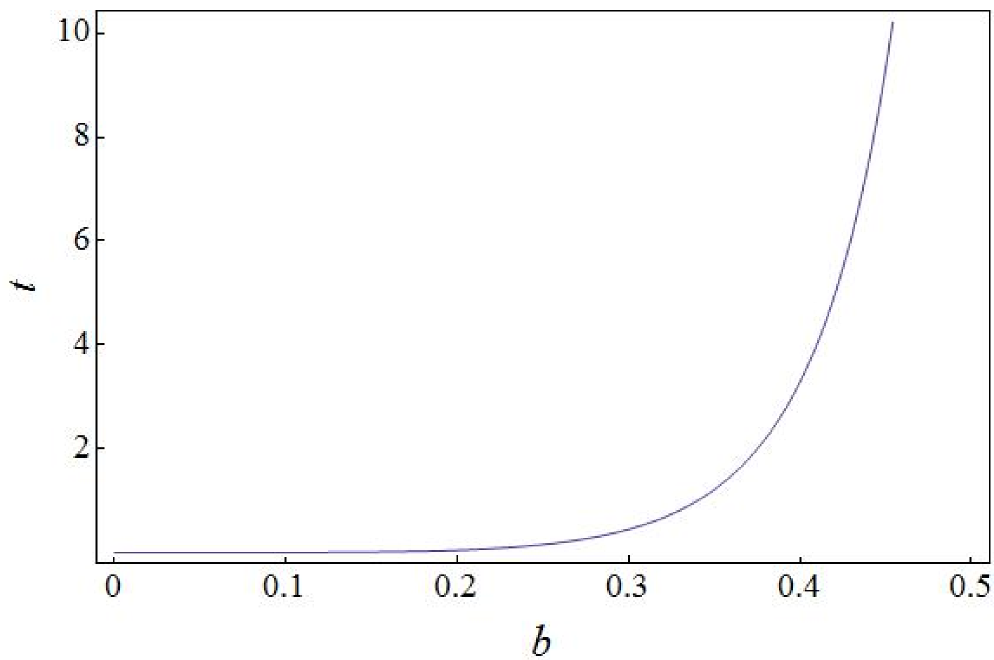
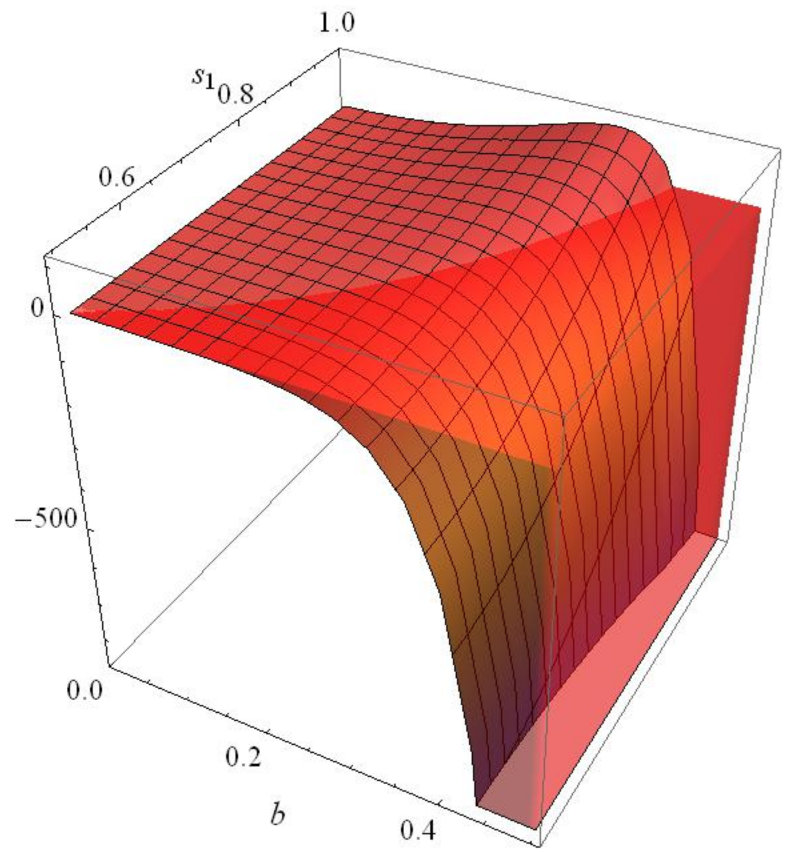


Figure 1 shows that the value of t necessary to set $2\pi_i(\mathbf{p}(w_p^*)) = \pi_M(\mathbf{p}^{\mathbf{M}_p})$ grows in b. This is because the more substitutable the products, the more able the JV to soften competition with firm 3 via w. In other words, the merger needs to have a relatively smaller synergy advantage to outperform the JV as products become more differentiated.

Asymmetric Ownership. Suppose that instead of splitting the ownership of the JV equally, firm 1 keeps $s_1 \in (1/2, 1]$ of the profits and firm 2 keeps $s_2 = 1 - s_1$. Moreover, suppose that firm 1 attains full control over w in exchange for a lump sum transfer to firm 2 that would leave both firms with equal expected profits. Finally, as in the previous subsection, suppose that firms compete in prices downstream and that demand is linear.

Depending on the timing of the lump sum transfer (or alternatively, the details of the JV contract), firm 1 may choose to set w to maximize its own profit, or alternatively, the joint profit of both JV partners. With this decision aside, the approach to solve for equilibrium prices, quantities, and profits follows the same approach as that used in Equations (A25) through Equations (A27). However, the resulting equations are highly unwieldy due to the presence of asymmetry. As such, for concision below, we set $a = 100$ and $k = 1$ and graph the relevant profits and prices while discussing the underlying intuition.[4]

Figure 2: $\pi_M(\mathbf{p}^{\mathbf{M}_p}) - \pi_1(\mathbf{p}(w_p^*)) - \pi_2(\mathbf{p}(w_p^*))$ for $b \in [0,\, 0.5)$ and $s_1 \in (1/2,\, 1]$



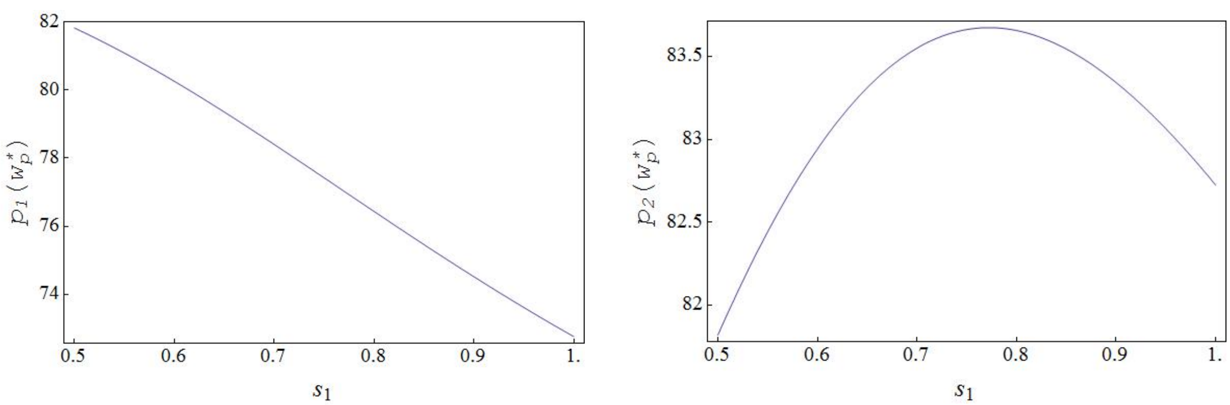
Suppose that firm 1 sets w to maximize joint profits. Figure 2 displays the difference between merger profits $\pi_M(\mathbf{p}^{\mathbf{M}_p})$ and the sum of JV partner profits $\pi_1(\mathbf{p}(w_p^*)) + \pi_2(\mathbf{p}(w_p^*))$

[4]Mathematica programs underlying the results are available upon request from the authors.

across all values of $b \in [0, 0.5)$ and $s_1 \in (1/2, 1]$. The opaque reddish region represents the part of the parameter space where the JV outperforms the merger. As the graph shows, a symmetric JV always outperforms the merger, but this becomes less likely as the ownership shares become more asymmetric. Moreover, the JV is more likely to outperform the merger for higher values of b. As mentioned above, this is because as products become more substitutable (b increases), the JV is better able to soften competition with firm 3.

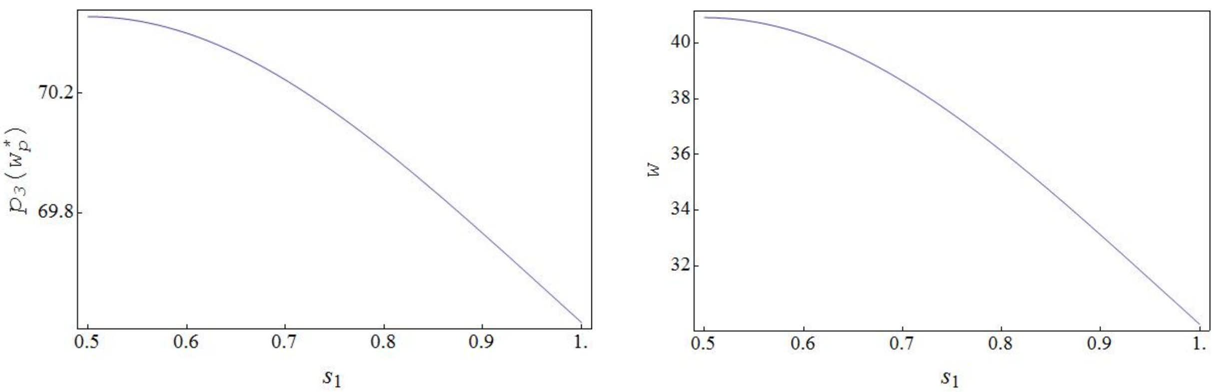
The intuition is as follows: Because of its higher ownership, as s_1 rises, firm 1 is induced to lower its downstream price (and w) relative to the symmetric case to raise demand for the input. Due to its lower ownership, firm 2 is induced to focus more on downstream profit and possibly to raise its downstream price, though the extent of that increase is constrained by firm 1's lower price (as is the price of firm 3). The culmination of this incentive misalignment can lead to lower average prices than following a symmetric JV and can result in lower cumulative profits than following a merger. We illustrate the effect of asymmetry on prices for $b = 1/4$ in Figures 3 and 4 below (3D graphs for $b \in [0, 0.5)$ are available upon request).

Figure 3: $p_1(w_p^*)$ and $p_2(w_p^*)$ for $a = 100$, $k = 1$, and $b = 1/4$ for $s_1 \in (1/2, 1]$



Observe that $p_1(w_p^*)$, $p_3(w_p^*)$, and w are decreasing as the ownership of the JV becomes more asymmetric. However, whereas $p_2(w_p^*)$ increases at first, there is a threshold at which firm 2's incentive to raise its downstream price in response to its lowered JV ownership due to an increase in s_1 is outweighed by decreases in the remaining downstream prices and the input price.

Figure 4: $p_3(w_p^*)$ and w for $a = 100$, $k = 1$, and $b = 1/4$ for $s_1 \in (1/2, 1]$

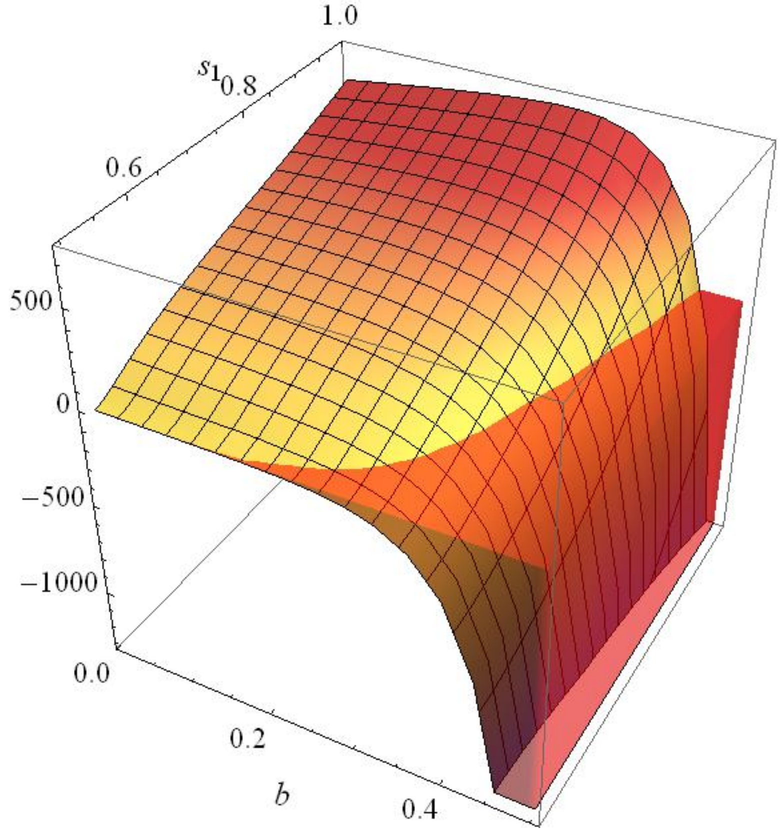


Next, suppose that firm 1 sets w to maximize own profit only. Looking at Figure 5, which represents a counterpart to Figure 2, when firm 1 sets the input price without regard to firm 2's profit, it should not be surprising that the size of the parameter space in which the JV outperforms the merger is now smaller. As before, a symmetric JV always outperforms the merger (this is difficult to visualize using Figure 5, but has already been proven in Proposition 4), but is less likely to do so as the ownership shares become more asymmetric and as b declines.

However, the direction of prices is different in this case. In particular, because firm 1 is no longer concerned about the impact of w on its partner's profit (except via the lump sum transfer), depending on the value of b, it may choose to raise w relative to the symmetric case to exploit its JV partner. This can lead all prices to rise as s_1 increases, though we note that firm 1's price is non-monotonic in s_1, and depending on the value of b, may end up above or below what it would be under symmetric ownership.[5]

[5] Additional graphs that display the relationships discussed in this paragraph are available upon request.

Figure 5: $\pi_M(\mathbf{p}^{\mathbf{M}_p}) - \pi_1(\mathbf{p}(w_p^*)) - \pi_2(\mathbf{p}(w_p^*))$ for $b \in [0, 0.5)$ and $s_1 \in (1/2, 1]$



A look at the curves that move along the s_1 axis in Figures 2 and 5 suggests that as s_1 goes from 1/2 to 1, $\pi_M(\mathbf{p}^{\mathbf{M}_p}) - \pi_1(\mathbf{p}(w_p^*)) - \pi_2(\mathbf{p}(w_p^*))$ increases. In other words, in the case of linear demand, it appears that the advantage that a JV has over a merger dissipates as the JV ownership becomes more asymmetric. This suggests that were the ownership of the JV an endogenous decision at the outset of our game, firms would prefer to organize by setting $s_1 = 1/2$, as in our baseline model.