Market Competition and the Internal Structure of Firms

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This paper investigates how the firm adjusts its internal structure and alters its incentive contracts in response to the market environment. Specifically, it is concerned with the firm’s optimal hierarchical structure and the corresponding incentive contracts for the managers as a function of variables which are related to the degree of market competition, when the middle manager’s sole function is information-gathering. Consistent with recent empirical literature, we show that under one measure of market competitiveness, an increase in competitiveness leads the firm to flatten its hierarchy and offer stronger incentives to its agents. However, under another measure, the reverse is true. The paper therefore not only offers a theoretical rationale for some of the recent empirical findings regarding the relation between market competition and the internal structure of firms, but also provides theoretical qualifications for these findings.

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1 Introduction

Recent empirical literature has shown that when the product market becomes more competitive, firms tend to flatten their hierarchies and offer

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stronger incentives to their employees. For example, using a panel-dataset of large U.S. firms from 1986 to 1999, Guadalupe and Wulf (2010) showed that competition leads to shallower hierarchies, broader managerial control, and increased incentives. While these empirical regularities are well established, there has been surprisingly little theoretical literature identifying the link between product market competition and internal firm structure.\footnote{For other empirical literature identifying similar regularities for various industries and countries, see Rajan and Wulf (2006), Cufat and Guadalupe (2009), Osterman (1996), and Whittington et al. (1999). For real-world cases, an example of increasing incentive payment as competition increases is Lincoln Electric (a company specializing in producing welding equipment) which, facing increasing global competition, increased total bonus payments from 35,000 in 2001 to 55,000 in 2005 (Siegel, 2007). An example of flattening hierarchy in response to competition is provided in Rajan and Wulf (2006). In 2002, GE, in response to increasing global competition, and in an attempt to speed up response to market changes, eliminated the position of Chairman of GE Capital. As a result, the managers in four departments under GE Capital had to report to the CEO directly.}

The purpose of the paper is to propose a simple theoretical model to capture and qualify these empirical regularities. Assume that a firm (the principal) hires two agents, whom we will call the division managers, to engage in production in two markets. For simplicity, assume that the result of production is binary: it either succeeds (which yields a high output) or fails (which yields a low output). Moreover, there exists market correlation (in the form of externalities) between the two markets. Therefore, production in the two markets requires coordination. The firm has the option to hire a third agent, who serves as the middle manager to (privately) observe the degree of externalities between the two markets before production. If it does so, then the firm has a three-tier hierarchy, with the firm providing incentives to the middle manager and the division managers. If the firm does not hire a middle manager, and offers contracts directly to the division manager in each market, it has a two-tier hierarchy.

In our model, the function of the middle-manager is to facilitate coordination of the production between the two markets. However, hiring a middle manager is also costly for the firm. More importantly, there is the well-known loss of control problem when the layers of hierarchy in the firm pile up.\footnote{We will discuss the theoretical literature later in this section.} Unlike the usual principal-agent model, which is solely concerned
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with the provision of incentives,\(^4\) or the organization theory, which is mainly concerned with the hierarchical structure of organization,\(^5\) in our model the firm has to solve for its hierarchical structure and the corresponding optimal contracts simultaneously.

The most challenging part in the attempt to link the theory and the results in the empirical literature is how to define the degree of competition. In Guadalupe and Wulf (2010), an increase in competition is taken as a result of the signing of the Canada-United States Free Trade Agreement of 1989, which liberalized trade between the two countries. In Rajan and Wulf (2006), a possible cause of increases in competition is the deregulation and increase in trade during 1986 and 1998. In Cuñat and Guadalupe (2009), the degree of competition is taken as import penetration into US as measured by changes in tariffs and exchange rates between 1992 and 2000. All these events have strong implication on competition, but also allow for a wide leeway in its interpretation. Our model proposes two measures of market competitiveness. Consistent with the empirical literature, we show that as the market becomes more competitive in the sense that a division manager’s efforts become less effective, it is more likely that a two-tier hierarchy will yield higher profit than a three-tier hierarchy. Moreover, a firm will also provide stronger incentives to its managers. However, if the market becomes more competitive in the sense that the firm’s revenue decreases even when production succeeds, then the conclusion is reversed: The firm will be more likely to adopt a three-tier hierarchy. Our results therefore not only provide a theoretical rationale for empirical results, but also give certain qualifications of what type of competition results in hierarchical delayering.

Recently, there has been an emerging theoretical literature which investigates the relation between market environment and organizational design. Schmidt (1997) investigates how product market competition affects the incentives of managers. He shows that competition increases the likelihood of liquidation on the one hand, and therefore a manager’s effort. On the other hand, since competition reduces a firm’s profit, it also weakens the manager’s effort. As a result, the relationship between competition and managerial incentives is non-monotonic. Raith (2003) shows that in an oligopolistic market, an increase of competition in the form of either increased product substitutability or a larger market will give a manager stronger incentive to

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\(^4\) See Prendergast (1999) for a survey of the literature.

\(^5\) See Radner (1992) for a survey.
reduce cost. Marin and Verdier (2008) investigates how a firm’s decision of whether to delegate formal power to low-level agents is affected by market competition. They show that there is a U-shaped relation between delegation and competition, with an intermediate level of competition resulting in the highest degree of delegation. Alonso et al. (2009) investigate how market competition affects a multi-division firm’s decision of whether to decentralize its decision-making. Based on the need to balance the tradeoff between adaptation to local market conditions and the distortion in information transmission, they show that if the cause of the increase in competition is lower demand, then it favors decentralization. If the cause is an increase in price elasticity, then it favors centralization. Rantakari (2008) investigates how market volatility affects the choice of a firm’s organizational structure. Consistent with the common wisdom, he shows that a stable environment favors an integrated and centralized structure, while a more volatile environment favors decentralization.

As mentioned earlier, our model incorporates both optimal contracting and hierarchical design decisions of the firm, therefore showing not only the effects of market competition on either hierarchical or incentive structure alone, but its effect on the two structures simultaneously. However, it is important to emphasize that the empirical regularities recorded in the literature, in particular Guadalupe and Wulf (2010), Rajan and Wulf (2006) and Cuñat and Guadalupe (2009), are rich and varied. Therefore, it is impossible to capture all these regularities in a model as simple as ours. In this paper, we are mainly concerned with the hierarchical structure and the incentive structure for the middle and division managers. The important issues in organizational theory regarding delegation, monitoring and control right allocation are beyond the scope of the paper, not because they are unimportant, but because we have not been able to successfully incorporate them into the model in a way that is clear and simple.

2 The Model

A firm is competing in two product markets, 1 and 2. Each market $i (i = 1, 2)$ is headed by a division manager $i$ who executes a project. The output of the project is denoted by $y_i$, which can be either high ($y_i = y^H$) or low ($y_i = y^L$, with $y^H > y^L > 0$). Given division manager $i$'s effort level $e_i \in [0, 1]$, the output of the project has a probability of $ae_i \equiv P(y^H|e_i)$ to be $y^H$, and a probability of $1 - ae_i \equiv P(y^L|e_i)$ to be $y^L$, where $0 < a < 1$. 
The value of $\alpha$ captures how the division manager’s effort translates into division performance. Later in the paper we will relate it to the degree of competition of the market. The expected output of division $i$ is therefore $\alpha e_i y^H + (1 - \alpha e_i) y^L$, and the marginal productivity of the division manager’s effort is $\alpha (y^H - y^L)$.

As in Alonso et al. (2008a,b), we assume that there exist externalities between the two divisions, whose effect on the division’s productivity might be either positive or negative. Formally, the firm’s profit from division $i$ is assumed to be

$$\pi_i(y_i, y_j | \phi) = \theta y_i + \phi y_j - w_i,$$

(1)

where $\theta$ is the revenue per unit of division $i$’s output, $\phi$ measures the external effect from the other divisions, and $w_i$ is the wage cost for hiring division manager $i$.\(^6\) As we allow for positive and negative externalities, for simplicity we assume that $\phi$ is a random variable which has a value of either $\epsilon > 0$ ($\theta > \epsilon$) with probability $1/2$, or $-\epsilon$ with probability $1/2$.\(^7\) Note that different realizations of $\phi$ will require different values of the division manager’s effort in order to maximize division profit.\(^8\)

The structure of the firm can be either a two-tier or three-tier hierarchy. In the former, the firm (which we will also call the “principal”) directly provides incentives to each of the division managers by offering them wage contracts. In the latter, the principal hires a middle manager whose function we will explain shortly. In this case the principal offers three incentive contracts, one for the middle manager, and one for each of the division managers. In both cases the principal designs contracts to maximize total profit, which is the sum of the two divisions’ profit minus total wage payments.

An important assumption in our model is that neither the principal nor the division managers can observe the value of $\phi$ until $y_i$’s are realized. Since knowing the value of $\phi$ can help the principal to internalize externalities by offering contracts to the division managers according to its true value, the main function of the middle manager in a three-tier hierarchy is to collect information for that purpose. We assume that by exerting an effort $e_m \in [0, 1]$, the middle manager can observe the value of $\phi$ with a probabil-

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\(^6\)For details of justifications for this setup, see Alonso et al. (2008a,b).

\(^7\)In order to make sure the division output is positive even if the externalities are negative, we assume that $\epsilon < \theta$.

\(^8\)From equation (1), it can be seen that a positive externality between the two markets ($\phi > 0$) implies a higher optimal effort level than the negative externality case ($\phi < 0$).
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He then reports its value to the principal, which in turn designs contracts for the division managers according to the report. However, information collection delays executing the project, so that hiring a middle manager has a negative effect on the efficiency of the project. Moreover, introducing a middle manager into the middle level of a hierarchy also creates a loss of control problem much discussed in the literature. We model the loss of control problem with two specifications. First, since the value of $\phi$ is unverifiable before the realization of $y_i$, the contract for the middle manager must induce them to tell the truth, which is a constraint on the efficiency of the incentive contract. Second, we assume that when the firm has a three-tier hierarchy, the probability of having high level output is $P(y^H|e_i) = \alpha \beta e_i$ where $0 < \beta < 1$. The value of $\beta$ is therefore a measure of the degree of loss of control.

All managers are identical and risk-averse, with utility function

$$U_a = u(w_a) - \frac{\sigma_a^2}{2},$$

where $w_a$ is the agent’s wage, with $a = 1, 2$ or $m$ when the agent is division manager 1, 2 or the middle manager, respectively; and

$$u(w_a) = \frac{w_a^{1-\rho}}{1-\rho},$$

where $\rho \in (0, 1)$ is the coefficient of relative risk aversion. The principal is risk-neutral and has a profit of $\sum_i \pi_i(y_i, y_j|\phi)$ in the two-tier hierarchy, and $\sum_i \pi_i(y_i, y_j|\phi) - w_m$ in the three-tier hierarchy.

The timing of events is as follows. First, the value of $\phi$ is realized, which is unobservable to either the principal or the division managers until $y_i$’s are

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9. Under our setup, it is the same whether the middle manager or the principal offers the contracts to the division managers. We therefore assume it is the latter, just for convenience of the proof later.

10. The middle manager actually also plays many other roles than information gathering. For example, his effort might increase the productivity of the division managers. In a three-tier hierarchy, this makes the optimal contract for the middle manager very difficult to solve. Therefore, we only emphasize the “figure out what to do” function of the middle manager, rather than its direct function in enhancing the division manager’s productivity. As can be seen from the model’s setup, we emphasize the control loss aspect of the middle manager in the production process.
realized. Then the principal makes the decision regarding the structure of the firm (two-tier or three-tier hierarchy). In the three-tier hierarchy where a middle manager is hired, the principal contracts with the middle manager first. After that, the middle manager chooses his effort level. Subsequent to this, he reports his observation to the principal, who in turn designs the contracts for the division managers. In the two-tier hierarchy case, the principal contracts to the division managers directly (without knowing the value of φ). Given the firm structure and the contracts, each division manager chooses an effort level. Finally, the revenue of each division (and therefore the value of φ) is revealed.

3 Optimal Contracts and Firm Structure

In this section, we first solve for the optimal contracts for the two-tier hierarchy case and the three-tier hierarchy case, respectively. Given the optimal contracts, we discuss the trade-off between the three-tier and the two-tier hierarchies.

3.1 The Optimal Contracts for the Two-tier Hierarchy

Since the division managers have no information about the externalities, they are not able to internalize the external effects. Moreover, making a division manager’s wage depend on the revenue of the other division cannot solve the externalities problem, as this only increases the risk borne by the division manager. Hence, it is optimal that the division manager’s wage depends only on his own division’s performance. This implies that the division manager $i$’s wage function is

$$w_i(y_i) = \begin{cases} w_i(y^H_i) \equiv w^H_i & \text{with probability } P(y^H_i|e_i), \\ w_i(y^L_i) \equiv w^L_i & \text{with probability } P(y^L_i|e_i). \end{cases}$$

The objective of the principal is to maximize his expected profit, subject to the individual rationality constraints, the incentive compatibility constraints and the limited liability constraints. Specifically, the maximization

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11 It can be easily shown that after the value of φ is realized, it does not matter whether the middle manager or the principal designs the contracts for the division managers.

12 The formal proof is provided in Appendix A.
problem can be written as

$$\max_{e_i, w_i} \sum_{y_1, y_2} P(y_1 | e_1) P(y_2 | e_2) [\pi_1(y_1, y_2 | \phi) + \pi_2(y_2, y_1 | \phi)]$$

s.t. IR$_i$: \(P(y_H | e_i) u(w_H^i) + P(y_L | e_i) u(w_L^i) - \frac{e_i^2}{2} \geq 0\),

IC$_i$: \(e_i \in \arg \max P(y_H | e_i) u(w_H^i) + P(y_L | e_i) u(w_L^i) - \frac{e_i^2}{2}\),

LL$_i$: \(w_H^i \geq 0\) and \(w_L^i \geq 0\),

where \(y_i = y^H, y^L\) and \(i = 1, 2\).

Since the two divisions are \textit{ex ante} identical, the optimal wage contracts are the same for the two division managers. Therefore, \(w_H^1 = w_H^2 \equiv w_H\), and \(w_L^1 = w_L^2 \equiv w_L\). We summarize the optimal contract in Lemma 1.

**Lemma 1.** The optimal contract for the division manager in the two-tier hierarchy case is: \(w_L = 0\) and

$$w_H = \left(\frac{1 - \rho}{2 - \rho}\right) \theta (y^H - y^L).$$

Moreover, the optimal effort level of the division manager is \(\alpha u(w_H) \equiv e_d^\ast\).

**Proof.** See Appendix B. \qed

Given the optimal contracts, the principal’s expected profit in the two-tier hierarchy is

$$\Pi^{2T} \equiv 2\theta y_L + 2\alpha^2 \left(\frac{u_H}{1 - \rho}\right)^{2-\rho}.$$  

### 3.2 The Optimal Contracts for the Three-tier Hierarchy

In the three-tier hierarchy case, the principal contracts to the middle manager in the first stage. After receiving the middle manager’s report about the\footnote{We focus on the interior solution equilibrium, which is ensured if

$$(\theta + \varepsilon) (y^H - y^L) < u^{-1}\left(\frac{1}{\alpha}\right) + \frac{1}{\alpha u}\left(\frac{1}{e_d}\right).$$}
value $\phi$, the principal designs the contracts for the division managers according to the report in the second stage. By backward induction, therefore, we solve for the optimal contracts for the division managers in the three-tier hierarchy first.

Given the middle manager’s report, $r$, the principal can design the contract for the division managers according to its value. Note that $r$ need only take three values: 0 (when the middle manager fails to observe the externalities), $\varepsilon$ and $-\varepsilon$. The objective of the principal is to maximize his expected profit, subject to the individual rationality constraints, the incentive compatibility constraints and the limited liability constraints. Specifically, the maximization problem is

$$\max_{e_i, w_i} \sum_{y_1, y_2} P(y_1|e_1)P(y_2|e_2) [\pi_1(y_1, y_2|r) + \pi_2(y_2, y_1|r)]$$

s.t. IR$_i$: $P(y^H|e_i)u(w^H_i) + P(y^L|e_i)u(w^L_i) - \frac{e_i^2}{2} \geq 0$,

IC$_i$: $e_i \in \arg\max P(y^H|e_i)u(w^H_i) + P(y^L|e_i)u(w^L_i) - \frac{e_i^2}{2}$,

LL$_i$: $w^H_i \geq 0$ and $w^L_i \geq 0$,

where $y_i = y^H, y^L$ and $i = 1, 2$.

We summarize the optimal contract for the division managers in the three-tier hierarchy case as follows.

**Lemma 2.** The optimal contract for the division manager is: $w^L_d(r) = 0 \forall r$, and

$$w^H_d(r) = \left(\frac{1 - \rho}{2 - \rho}\right) (\theta + r) (y^H - y^L).$$

Moreover, the optimal effort level of the division managers is $\equiv \alpha\beta u(w^H_d(r)) \equiv e^*_d(r)$.

**Proof.** See Appendix B. 

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14 Please see Appendix B, where we prove that when the principal does not know the value of $\phi$ (i.e., when the middle manager reports that he does not observe $\phi$), it is optimal for the principal to offer contracts to the division managers conditional on $\phi$ being its expected value, which is 0. Also note that $\phi$ can never take the value of 0. Therefore, when the middle manager reports 0, it is equivalent to reporting he does not observe its value.
The expected profit of the firm when the middle manager successfully observes $\phi$ is

$$\pi^1 = 2\theta y^L + 2(\alpha\beta)^2 \left\{ \frac{\left[w^H_H(\epsilon)\right]^{2-\rho}}{2(1-\rho)^2} + \frac{\left[w^H_H(-\epsilon)\right]^{2-\rho}}{2(1-\rho)^2} \right\};$$

and is

$$\pi^\emptyset = 2\theta y^L + 2\alpha\beta \frac{\left(w^H_H(0)\right)^{2-\rho}}{(1-\rho)^2}$$

if the middle manager fails to observe it. Define $\Delta \pi = \pi^1 - \pi^\emptyset$ as the benefit of coordinating the externalities between the two divisions.

We now turn to the optimal contract for the middle managers in the first stage. Note that in our model, the middle manager’s effort can increase the probability of observing $\phi$, but not the probability of the project’s success. Therefore, there is no need for the middle manager’s wage to depend on the division outputs, for this only increases the risk of the middle manager’s wage income without improving the probability of the division’s project succeeding. In Appendix C, we prove that the optimal contract for the middle manager is independent of division performance.

By our specification, the middle manager can either observe the true value of $\phi$ or nothing. Moreover, the value of $\phi$ can be verified ex post. Finally, if the middle manager does not observe the value of $\phi$, but intends to game on a report, his best guess is the mean value of $\phi$, i.e., to report $\phi = 0$, but in that case it is like announcing that he does not observe $\phi$, as its true value is never 0. Therefore, the middle manager’s wage needs only depend on whether he has reported observing $\phi$ (i.e., whether $r = 0$) and, if yes, whether his report is correct. (See Appendix C for formal proof.) That is, there only need be three levels of wage for the middle manager: when he fails to observe ($r = 0$), when he reports correctly ($r = \phi$), and when he reports incorrectly ($r \neq 0, r \neq \phi$). Specifically, the middle manager’s wage is

$$w_m(r) = \begin{cases} 
    w^\emptyset_m & \text{if } r = 0, \\
    w^\phi_m & \text{if } r = \phi, \\
    w^0_m & \text{otherwise.} 
\end{cases} \quad (2)$$

In addition to the individual rationality constraint, the incentive compatibility constraint and the limited liability constraint, the principal’s profit...
maximization problem is also subject to the middle manager’s truthful-revelation constraints, which require that, first, if $\phi$ is observed by the middle manager, he will report $r = \phi$ rather than report $r \neq \phi$ or $r = 0$. That is,

$$u (w^1_m) \geq u (w^0_m) \quad \text{and} \quad u (w^1_m) \geq u (w^0_m).$$

Second, if the middle manager does not observe $\phi$, he will not be willing to game it by reporting an arbitrary value. Specifically,

$$u (w^0_m) \geq \frac{1}{2} u (w^1_m) + \frac{1}{2} u (w^0_m).$$

Given the optimal contract for the division managers, the objective of the principal in the first stage is therefore to maximize the expected profit, subject to the individual rationality constraint, the incentive compatibility constraint, limited liability constraint and the truthful-revelation constraints. Note that the middle manager’s contract will induce his truthful report. Therefore, when the middle manager does not observe the value of $\phi$, he will report as such, and the optimal contract for the division managers will be exactly the same as in Lemma 1. The profit maximization problem can be written as

$$\max_{e_m, w_m} e_m \left( \pi^1 - w^1_m \right) + (1 - e_m) \left( \pi^0 - w^0_m \right)$$

subject to

IR$_m$: $e_m u (w^1_m) + (1 - e_m) u (w^0_m) - \frac{1}{2} e_m^2 \geq 0,$

IC$_m$: $e_m \in \arg \max e_m u (w^1_m) + (1 - e_m) u (w^0_m) - \frac{1}{2} e_m^2,$

LL$_m$: $w^1_m \geq 0$, $w^0_m \geq 0$ and $w^0_m \geq 0$,

TR$_1$: $u (w^1_m) \geq u (w^0_m),$

TR$_2$: $u (w^1_m) \geq u (w^0_m),$

TR$_3$: $u (w^0_m) \geq \frac{1}{2} u (w^1_m) + \frac{1}{2} u (w^0_m).$

The equilibrium contract for the middle manager is characterized in the following lemma.15

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15The condition for an interior solution $e_m < 1$ is $\pi^1 - \pi^0 < u^{-1}(2) + (2/u'(u^{-1}(2))) - u^{-1}(1)$. 

Lemma 3. Under the optimal contract for the middle manager, \( w_m^0 = 0 \), \( w_m^\emptyset \) satisfies

\[
\frac{2 - \rho}{1 - \rho} \left( 2^{1/\tau} - 1 \right) w_m^\emptyset + (w_m^\emptyset)\rho = \Delta \pi,
\]

and \( w_m^1 = u^{-1}(2u(w_m^\emptyset)) \). Moreover, \( e_m = u(w_m^\emptyset) \). The principal’s expected profit in the three-tier hierarchy is therefore

\[
\Pi^{3T} = u(w_m^\emptyset) \left( \Delta \pi - w_m^1 + w_m^\emptyset \right) + \pi^\emptyset - w_m^0.
\]

Proof. See Appendix D.

Lemma 3 implies that both \( w_m^1 \) and \( w_m^\emptyset \) are increasing functions of \( \Delta \pi \).\footnote{See Appendix D for the proof.} The reason for this is, first, a greater value of \( \Delta \pi \) means that learning the value of \( \phi \) is more valuable. Therefore, the principal is willing to increase \( w_m^1 \) in order to induce the middle manager’s higher effort. (Note that \( w_m^0 = 0 \).) However, a higher value of \( w_m^1 \) increases the middle manager’s incentive to risk reporting either \( \epsilon \) or \( -\epsilon \) when he does not observe the value of \( \phi \). In order to maintain the truthful-revelation incentive, the principal also has to raise \( w_m^\emptyset \) as \( \Delta \pi \) increases.

3.3 Firm Structure

This section discusses how the changes in market conditions affect the relative profit of the firm under the two-tier and three-tier hierarchies.

There are two possible measures of market competitiveness in our model. First, recall that \( \theta \) is how much a unit of output is translated into division revenue. A reduction in its value means a lower profit margin. We will therefore say that the market becomes more competitive when there is a decrease in the value of \( \theta \). Second, as has been shown, a decrease in the value of \( \alpha \) implies a reduction in the division manager’s productivity. Therefore, we can also interpret a reduction in the value of \( \alpha \) as an increase in market competitiveness.

Let \( B = \Pi^{2T} - \Pi^{3T} \) be the difference in total profits under the two-tier and the three-tier hierarchies. \( B \) can be rewritten into the following form:

\[
B = \Pi^{2T} - \left\{ u \left( w_m^\emptyset \right) \pi^1 + \left[ 1 - u \left( w_m^\emptyset \right) \right] \pi^\emptyset \right\} - u \left( w_m^\emptyset \right) w_m^1 - \left[ 1 - u \left( w_m^\emptyset \right) \right] w_m^\emptyset.
\]
Note that if the loss of control is serious (i.e., $\beta$ is smaller than a value $\bar{\beta}$ for the determination of its value; please see the proof for Proposition 1) so that $u(w^\emptyset_m)\pi^1 + [1 - u(w^\emptyset_m)]\pi^\emptyset$ is smaller than $\Pi^{2T}$, then the three-tier hierarchy’s profit is always lower than the two-tier hierarchy. In that case, the principal always prefers a two-tier to three-tier hierarchy. We will therefore focus on the more interesting case of when $\beta \geq \bar{\beta}$. In this case, the choice between a two-tier and three-tier hierarchy is to balance the benefit of internalizing the externalities between the two divisions (by observing $\phi$) and the costs of loss of control (in the form of reducing the probability of success by a factor $\beta$) and the middle manager’s wage.

**Proposition 1.** The benefit for the two-tier hierarchy relative to the three-tier hierarchy is increasing in the firm’s profit per unit of output (that is, $\partial B/\partial \theta > 0$) and, if $\beta > \bar{\beta}$, decreasing in the productivity of the division manager, that is, $\partial B/\partial \alpha < 0$.

**Proof.** See Appendix E. \(\square\)

To grasp the intuition behind Proposition 1, first note that the effects from a change of $\theta$ and $\alpha$ on $B$ can be derived as

$$\frac{\partial B}{\partial \theta} = \frac{\partial}{\partial \theta} \left( \Pi^{2T} - \pi^\emptyset \right) - u \left( w^\emptyset_m \right) \frac{\partial \Delta \pi}{\partial \theta},$$

$$\frac{\partial B}{\partial \alpha} = \frac{\partial}{\partial \alpha} \left( \Pi^{2T} - \pi^\emptyset \right) - u \left( w^\emptyset_m \right) \frac{\partial \Delta \pi}{\partial \alpha}.$$

Recall that $\pi^\phi$ is the firm’s profit when the middle manager fails to observe $\phi$. Therefore, the first terms on the right-hand side of both equations above are the changes in the cost of the loss of control (resulting from hiring a middle manager) when $\theta$, $\alpha$ increases. The second terms in both equations are the changes in the benefit of internalizing the externalities between the two divisions. Lemmas 1 and 2 imply that the division managers’ efforts are increasing in both $\alpha$ and $\theta$. Since the increasing effort of the division managers will enlarge the effect of loss of control, we know that

$$\frac{\partial}{\partial \theta} \left( \Pi^{2T} - \pi^\emptyset \right) > 0 \text{ and } \frac{\partial}{\partial \alpha} \left( \Pi^{2T} - \pi^\emptyset \right) > 0.$$

Also recall that $\Delta \pi$ is the benefit of observing $\phi$. Moreover, the profit of the division $i$ is the sum of $\theta y_i$ and $\phi y_j$ minus the division managers’ wage.
Since an increase in $\theta$ makes observing $\phi$ less important, we know that $\partial \Delta \pi / \partial \theta < 0$. As a consequence, we have $\partial B / \partial \theta > 0$ in Proposition 1. On the other hand, an increase in $\alpha$ will result in an increase in the expected output. Therefore, the benefit of internalizing the externalities is also greater, which implies $\partial \Delta \pi / \partial \alpha > 0$. When the loss of control is not too serious ($\beta \geq \bar{\beta}$), the enlarged effect of loss of control will be dominated by the benefit of internalizing the externalities. Therefore, we have $\partial B / \partial \alpha < 0$ in Proposition 1.

4 Incentive Provision

Empirical papers, such as Cuñat and Guadalupe (2009) and Guadalupe and Wulf (2010), have found that the incentive provided to the manager has a positive relationship with market competition. In our paper, we show that, consistent with the findings of both papers, stronger market competition results in a stronger incentive provision. We also show that a flatter firm structure may arise at the same time.

We first need to define the power of incentive by the wage contract. In the empirical papers, the power of incentive is measured by performance-based payment. Since $w_d^L(r) = 0$, the contract’s power of incentive can be measured directly by the expected wage payment which, by Lemma 1 and Lemma 2, is

$$P \left( y^H | e_d^* \right) w_d^H(r) + \left[ 1 - P \left( y^H | e_d^* \right) \right] w_d^L(r).$$

In the two-tier hierarchy case, this equals to

$$W_{2T} = \alpha^2 \left[ w_d^H(0) \right]^{2-\rho} \frac{1}{1-\rho}.$$

In the three-tier hierarchy case, this is equal to

$$W_{3T} = \alpha^2 \left( \frac{w_m^0}{2(1-\beta)} \right)^2 \left[ \frac{w_d^H(\epsilon)}{2(1-\rho)} \right]^{2-\rho} + \left[ 1 - u(w_m^0) \right] (\alpha \beta)^2 \frac{w_d^H(0)}{1-\rho}.$$

As mentioned earlier, since $w_d^L(r) = 0$, the powers of the incentives in the two-tier and three-tier hierarchy cases are actually the values of $W_{2T}$ and $W_{3T}$, respectively.
Proposition 2. If \( B \geq 0 \), then the incentive provided in the two-tier hierarchy case is stronger. That is, \( \mathcal{W}_{2T} - \mathcal{W}_{3T} > 0 \).

Proof. It is straightforward to see that

\[
\mathcal{W}_{2T} - \mathcal{W}_{3T} = \frac{1}{2} \left\{ (1 - \rho) B + u \left( w_{m}^{\bar{B}} \right) w_{m}^{1} + \left[ 1 - u \left( w_{m}^{\bar{B}} \right) \right] w_{m}^{0} \right\}.
\]

From Lemma 3, \( w_{m}^{1} > 0 \), \( w_{m}^{0} > 0 \), and \( u(w_{m}^{\bar{B}}) \in (0, 1) \). Therefore \( \mathcal{W}_{2T} - \mathcal{W}_{3T} > 0 \) when \( B \geq 0 \).

The intuition of Proposition 2 is as follows. In the two-tier hierarchy, there is no project delay, and thus the division managers’ efforts are more effective. As a result, the owner of the firm is more willing to increase the division manager’s effort than in the three-tier hierarchy. To implement a higher effort level, the owner needs to pay a larger wage to the division managers if outputs are high. This explains the result in Proposition 2.

Proposition 2 is very useful for explaining the empirical findings. In particular, Cufat and Guadalupe (2009) and Guadalupe and Wulf (2010) had identified a positive link between the degree of competition and the power of incentive provided by the contract. Guadalupe and Wulf (2010) further found the coexistence of the changes in firm structure and the strength of incentive. From Proposition 1 we know that, if \( \beta > \bar{\beta} \), competition (in the form of a decrease in the division manager’s productivity) will tend to favor the two-tier hierarchy relative to three-tier hierarchy. Moreover, from Proposition 2 we know that the division manager’s incentive is strengthened whenever the firm structure is flattened. Therefore, the firm responds to stronger competition by delayering the hierarchy and offering stronger incentives, as the following corollary summarizes.

Corollary 1. If competition reduces the division manager’s productivity (\( \alpha \) is smaller) and causes the principal to flatten the hierarchy, then the incentives provided to the managers are stronger.

We hasten to add that not all competition leads to delayering. As can be seen from Proposition 1, competition in the form of a reduction in revenue per unit of output actually causes the firm to deepen its hierarchy. In the empirical literature, Cufat and Guadalupe (2009) showed that an increase in the volume of imports in the previous period positively affects incentives provision by the firm, and impels the firm to flatten its hierarchy in the next
period. Guadalupe and Wulf (2010) showed the same effects after liberalization of trade between the US and Canada. There is no doubt that both the increase in imports and liberalization of trade increase market competition. However, an increase in competition can mean many things, and we have yet to pin down precisely which ingredients affect the firm’s decision on their internal structures. For example, an increase in imports might intensify market competition through a decrease in prices, increase in the number of firms, or increase in product variety. Similarly, trade liberalization might increase competition in such a way that the firms have lower probability of succeeding (i.e., $\alpha_e$ is lower in our model), but higher output when they do succeed ($y^H$ is higher in our model). However, it can also simply mean that revenue is lower for each unit of output (a lower $\theta$ in our model). Without closer inspection, we do not know exactly which is the driving force behind the firm’s decision on internal structure. Our results therefore call for a more detailed analysis of what factors influence market competition, and what impact each one is.

5 Conclusion

In this paper, we propose a simple theoretical model to explain and qualify the empirical regularities recorded in the recent empirical literature regarding market competition and firm structure. Specifically, if market competition reduces the division manager’s productivity, the firm will respond with a flatter hierarchy and provide stronger incentives to its managers. An interesting extension is to endogenize the numbers of the hierarchy’s layers and the agents. In our model, since there can be only one middle manager and two division managers, the scope of control of the firm necessarily broadens (from 1 to 2) when the middle manager’s level is deleted. In a general context when the numbers of the middle and division managers are also choice variables, we expect much more complicated, but far more rewarding, theoretical results to emerge.

Appendix

A. The Wage Contracts for the Division Managers

Assume that the wage contracts in the two-tier hierarchy are contingent on both divisions’ profits. Specifically, the wages are
The maximization problem can be written as

\[
\begin{align*}
    w_i(y_i, y_j) &= \begin{cases} 
        w_i(y_i, y_j) & \text{if } i = j, \\
        w_i(y_i, y_j) & \text{if } i \neq j,
    \end{cases}
\end{align*}
\]

where \( i \neq j, i, j \in \{1, 2\}. \) Let \( z \) be the principal’s information regarding \( \phi. \) Given the definition of division profit (equation (1)), the principal’s profit maximization problem can be written as

\[
\begin{align*}
    \max_{\epsilon_i, w_i} E_{y, \phi \pi} &= (\theta + E(\phi | z)) \left[ \alpha e_i \alpha e_2 (y_i^H + y_i^H) \\
    &+ (1 - \alpha e_i)(1 - \alpha e_2) (y_i^L + y_j^L) \right] \\
    &- \left[ \alpha e_i \alpha e_2 (w_i^H + w_2^H) + \alpha e_1 (1 - \alpha e_2) (w_i^L + w_2^L) \right] \\
    &+ (1 - \alpha e_i)(1 - \alpha e_2) \left[ w_i^H + w_2^H \right] \right] \\
\text{s.t.} \quad & \text{IR}'_i: \alpha e_2 \left[ \alpha e_i u \left( w_i^H \right) + (1 - \alpha e_j) u \left( w_i^L \right) \right] \\
&+ (1 - \alpha e_i)(1 - \alpha e_j) u \left( w_i^L \right) \geq \frac{\Theta^2}{2}, \\
& \text{IC}'_i: \alpha \left[ \alpha e_i u \left( w_i^H \right) + (1 - \alpha e_j) u \left( w_i^L \right) \right] - \alpha \left[ \alpha e_i u \left( w_i^H \right) \\
&+ (1 - \alpha e_j) u \left( w_i^L \right) \right], \\
& \text{LL}'_i: w_i^H \geq 0, w_i^L \geq 0, w_i^L \geq 0 \text{ and } w_i^L \geq 0.
\end{align*}
\]

We will prove it cannot be optimal if \( w_i^H \neq w_i^H \) and \( w_i^L \neq w_i^L \). For if so, suppose a wage schedule \( \hat{w}_i^H \neq \hat{w}_i^H \) and \( \hat{w}_i^L \neq \hat{w}_i^L \) solved the principal’s profit maximization problem. Since the agents are risk-averse, we can find a wage schedule \( \tilde{w}_i^H \equiv \tilde{w}_i^H = \hat{w}_i^H \) such that \( u(\tilde{w}_i^H) = \alpha e_i u(\hat{w}_i^H) + (1 - \alpha e_j) u(\hat{w}_i^L) \) and \( \tilde{w}_i^L < \alpha e_j \tilde{w}_i^H + (1 - \alpha e_j) \tilde{w}_i^H \). Similarly, we also can find a wage schedule \( \tilde{w}_i^L \equiv \tilde{w}_i^L = \hat{w}_i^L \) such that \( u(\tilde{w}_i^L) = \alpha e_i u(\hat{w}_i^L) + (1 - \alpha e_j) u(\hat{w}_i^L) \) and \( \tilde{w}_i^L < \alpha e_j \tilde{w}_i^L + (1 - \alpha e_j) \tilde{w}_i^L \). Obviously, the new contract \( (\tilde{w}_i^H, \tilde{w}_i^L) \) satisfies all the constraints of the problem. Furthermore, the principal can implement as much effort as \( \tilde{w}_i^H \neq \tilde{w}_i^H \) and \( \tilde{w}_i^L \neq \tilde{w}_i^L \) with lower wage cost. Hence, the contract is optimal only if \( w_i^H = w_i^H \) and \( w_i^L = w_i^L \).
B. The Optimal Wage Contract for the Division Managers

Let \( z \) be the principal’s information regarding \( \phi \). Since \( P(y^H|e_i) = 1 - P(y^L|e_i) \), \( \partial P(y^H|e_i)/\partial e_i = -(\partial P(y^L|e_i)/\partial e_i) \equiv P_e \), the Lagrangian function of the profit maximization problem can be written as

\[
L_d = \sum_{y_1, y_2} P(y_1|e_1) P(y_2|e_2) [\pi_1(y_1, y_2|z) + \pi_2(y_2, y_1|z)]
\]

\[
+ \sum_{i=1}^{2} \lambda_i \left[ P(y^H|e_i)u(w^H_i) + P(y^L|e_i)u(w^L_i) - \frac{e_i^2}{2} \right]
\]

\[
+ \sum_{i=1}^{2} \lambda_{i+2} [P_e u(w^H_i) - P_e u(w^L_i) - e_i]
\]

\[
+ \sum_{i=1}^{2} \lambda_{i+4} w^H_i + \sum_{i=1}^{2} \lambda_{i+6} w^L_i.
\]

Substituting the IC into the IR constraint yields \( u(w^L_i) + e_i^2[(P(y^H|e_i)/P_e e_i) - (1/2)] \geq 0 \). Since \( (P(y^H|e_i)/P_e e_i) = 1 \), the IR constraints are non-binding (i.e., \( \lambda_i = 0 \)). From the first-order conditions, we have

\[
\frac{\partial L_d}{\partial e_i} = 0 \Rightarrow P_e \left[ (\theta + z) (y^H - y^L) - (w^H_i - w^L_i) \right] = \lambda_{i+2}. \tag{B.1}
\]

\[
\frac{\partial L_d}{\partial w^H_i} = 0 \Rightarrow \lambda_{i+2} P_e u'(w^H_i) + \lambda_{i+4} = P(y^H|e_i), \tag{B.2}
\]

\[
\frac{\partial L_d}{\partial w^L_i} = 0 \Rightarrow -\lambda_{i+2} P_e u'(w^L_i) + \lambda_{i+6} = 1 - P(y^H|e_i). \tag{B.3}
\]

From B.3, we have \( \lambda_{i+6} = 1 - P(y^H|e_i) + \lambda_{i+2} P_e u'(w^L_i) > 0 \), which implies that \( w^L_i = 0 \) and \( e_i = P_e u(w^H_i) \). Since the IR constraints are non-binding, \( w^L_i = 0 \) implies that \( w^H_i > 0 \). Therefore, we have \( \lambda_{i+4} = 0 \). Hence, the CRRA assumption, B.1 and B.2 imply that

\[
w^H_i(z) = \left( \frac{1-\rho}{2-\rho} \right) (\theta + z) (y^H - y^L). \tag{B.4}
\]

The optimal contract is characterized by \( w^L_i = 0 \) and B.4.

In what follows, we prove that it is optimal for the principal to take \( z \) as its expected value, 0, when he has no information regarding \( \phi \). The principal’s profit function is
\[ \pi(z|\phi) = (\theta + \phi) y^L + P \left( y^H | e^*_d \right) \left[ (\theta + \phi) (y^H - y^L) - w^H_d(z) \right]. \]

If \( \phi \) is not observed, the principal knows that \( \phi = \varepsilon \) or \( \phi = -\varepsilon \) with equal probability and, therefore,

\[ E_\phi \pi(z|\phi) = \frac{1}{2} \pi(z|\varepsilon) + \frac{1}{2} \pi(z|-\varepsilon). \]

Knowing that, \( z \) maximizes the expected profit only if

\[ \frac{\partial E_\phi \pi(z|\phi)}{\partial z} = -z P_e u'(w^H(z)) (y^H - y^L) = 0. \]

Since \( P_e > 0, u'(w^H(z)) > 0 \) and \( y^H - y^L > 0 \), this (first-order) condition holds if and only if \( z = 0 \), which is exactly the expected value of \( \varepsilon \) (or \( \varepsilon \)). Note that the second-order condition is

\[ \frac{\partial^2 E_\phi \pi(z|\phi)}{(\partial z)^2} \bigg|_{z=0} = -P_e u'(w^H(z)) (y^H - y^L) < 0. \]

Therefore, we prove that when \( \phi \) is not observed, it is optimal for the principal to offer the contract to the division managers conditional on the value of \( z \) being 0.

C. The Wage Contract for the Middle Manager

We will show that it is not optimal for the middle manager’s wage contract to be contingent on realized sales.

Let the wages for the middle manager, when \( r = \phi \) and \( r = 0 \), respectively, be

\[
\begin{align*}
    w^H_m(y_1, y_2) &= \begin{cases} 
w^{HH}_m & \text{if } y_1 = y^H \text{ and } y_2 = y^H; \\
w^{HL}_m & \text{if } y_1 = y^H \text{ and } y_2 = y^L; \\
w^{LH}_m & \text{if } y_1 = y^L \text{ and } y_2 = y^H; \\
w^{LL}_m & \text{if } y_1 = y^L \text{ and } y_2 = y^L,
\end{cases} \\
    w^L_m(y_1, y_2) &= \begin{cases} 
w^{HH}_m & \text{if } y_1 = y^H \text{ and } y_2 = y^H; \\
w^{HL}_m & \text{if } y_1 = y^H \text{ and } y_2 = y^L; \\
w^{LH}_m & \text{if } y_1 = y^L \text{ and } y_2 = y^H; \\
w^{LL}_m & \text{if } y_1 = y^L \text{ and } y_2 = y^L,
\end{cases}
\]

and
Let the profit for the principal, when \( r = \phi \) and \( r = 0 \), respectively, be

\[
\pi^1(y_1, y_2) = \begin{cases} 
\pi_{HH}^1 & \text{if } y_1 = y^H \text{ and } y_2 = y^H; \\
\pi_{HL}^1 & \text{if } y_1 = y^H \text{ and } y_2 = y^L; \\
\pi_{LH}^1 & \text{if } y_1 = y^L \text{ and } y_2 = y^H; \\
\pi_{LL}^1 & \text{if } y_1 = y^L \text{ and } y_2 = y^L,
\end{cases}
\]

and

\[
\pi^\emptyset(y_1, y_2) = \begin{cases} 
\pi_{HH}^\emptyset & \text{if } y_1 = y^H \text{ and } y_2 = y^H; \\
\pi_{HL}^\emptyset & \text{if } y_1 = y^H \text{ and } y_2 = y^L; \\
\pi_{LH}^\emptyset & \text{if } y_1 = y^L \text{ and } y_2 = y^H; \\
\pi_{LL}^\emptyset & \text{if } y_1 = y^L \text{ and } y_2 = y^L,
\end{cases}
\]

To simplify notations, let

\[
\begin{align*}
\alpha e_1 \alpha e_2 u_{HH}^1 + & \alpha e_1 (1 - \alpha e_2) u_{HL}^1 + (1 - \alpha e_1) \alpha e_2 u_{HH}^1 \\
+ & (1 - \alpha e_1)(1 - \alpha e_2) u_{HL}^1 = \Omega^1, \\
\alpha e_1 \alpha e_2 u_{HH}^\emptyset + & \alpha e_1 (1 - \alpha e_2) u_{HL}^\emptyset + (1 - \alpha e_1) \alpha e_2 u_{HH}^\emptyset \\
+ & (1 - \alpha e_1)(1 - \alpha e_2) u_{HL}^\emptyset = \Omega^\emptyset, \\
\alpha e_1 \alpha e_2 u (u_{HH}^1) + & \alpha e_1 (1 - \alpha e_2) u (u_{HL}^1) + (1 - \alpha e_1) \alpha e_2 u (u_{HL}^1) \\
+ & (1 - \alpha e_1)(1 - \alpha e_2) u (u_{HL}^1) = \Psi^1, \text{ and} \\
\alpha e_1 \alpha e_2 u (u_{HH}^\emptyset) + & \alpha e_1 (1 - \alpha e_2) u (u_{HL}^\emptyset) + (1 - \alpha e_1) \alpha e_2 u (u_{HL}^\emptyset) \\
+ & (1 - \alpha e_1)(1 - \alpha e_2) u (u_{HL}^\emptyset) = \Psi^\emptyset.
\end{align*}
\]

The profit maximization problem is then

\[
\max_{e_m, u_m} \left[ \alpha e_1 \alpha e_2 \pi_{HH}^1 + \alpha e_1 (1 - \alpha e_2) \pi_{HL}^1 + (1 - \alpha e_1) \alpha e_2 \pi_{HH}^1 \\
+ (1 - \alpha e_1)(1 - \alpha e_2) \pi_{HL}^1 - \Omega^1 \right] + (1 - e_m) \left[ \alpha e_1 \alpha e_2 \pi_{HH}^\emptyset \\
+ \alpha e_1 (1 - \alpha e_2) \pi_{HL}^\emptyset + (1 - \alpha e_1) \alpha e_2 \pi_{HH}^\emptyset \\
+ (1 - \alpha e_1)(1 - \alpha e_2) \pi_{HL}^\emptyset - \Omega^\emptyset \right]
\]

s.t. \( IR_m' : e_m \Psi^1 + (1 - e_m) \Psi^\emptyset \geq \frac{e_m^2}{2} \),

\( IC_m' : \Psi^1 - \Psi^\emptyset = e_m \),

\( LL_m' : u_{HH}^1 \geq 0, u_{HL}^1 \geq 0, u_{HH}^1 \leq 0, u_{HL}^1 \leq 0, u_{HH}^\emptyset \geq 0, u_{HL}^\emptyset \geq 0, u_{HL}^1 \leq 0, u_{HL}^\emptyset \leq 0, u_{HL}^1 \geq 0, u_{HL}^\emptyset \geq 0, u_{HL}^1 \leq 0, u_{HL}^\emptyset \leq 0. \)
Market Competition and the Internal Structure of Firms

TR1: $\Psi^1 \geq u \left( w_m^0 \right)$.

TR2: $\Psi^1 \geq \Psi^0$.

TR3: $\Psi^0 \geq \frac{1}{2} \Psi^1 + \frac{1}{2} u \left( w_m^0 \right)$.

Assume there is a wage schedule such that $(w_m^{HH1}, w_m^{HL1}, w_m^{LH1}, w_m^{LL1})$ are not all equal or $(w_m^{HH0}, w_m^{HL0}, w_m^{LH0}, w_m^{LL0})$ are not all equal. Furthermore, suppose this wage schedule solves the principal’s profit maximization problem. Since the middle manager is risk-averse, we can find a wage schedule $\hat{w}^1_m$ such that $u(\hat{w}^1_m) = \Psi^1$ and $\hat{w}^1_m < \Omega^1$. Similarly, we can find a new wage schedule so that $\hat{w}^0_m$ satisfies all the constraints.

Obviously, the new wage schedule $(\hat{w}^1_m, \hat{w}^0_m)$ satisfies all the constraints. Furthermore, the principal can implement the same effort level with a lower wage cost. Therefore, the wage schedule $(w_m^{HH1}, w_m^{HL1}, w_m^{LH1}, w_m^{LL1})$ and $(w_m^{HH0}, w_m^{HL0}, w_m^{LH0}, w_m^{LL0})$ can never be optimal. The intuition of this proof is simple: the contract which assigns a different wage to the middle manager not only adds no information value for solving the moral hazard problem, but also increases the expected wage cost.

D. Proof of Lemma 3

From Lemmas 1 and 2, we have

$$\pi^1 = 2\theta y^L + 2(\alpha\beta)^2 \left\{ \frac{\left[ w_H^0 (\varepsilon) \right]^{2-\rho}}{2(1-\rho)^2} + \frac{\left[ w_H^0 (-\varepsilon) \right]^{2-\rho}}{2(1-\rho)^2} \right\},$$

$$\pi^0 = 2\theta y^L + 2(\alpha\beta)^2 \left[ \frac{w_H^0 (0)^{2-\rho}}{(1-\rho)^2} \right].$$

Because $\rho < 1$, we know that $[u(w_H^0(z))]^{2-\rho}$ is convex in $z$ and therefore $\pi^1 - \pi^0 \geq 0$.

The Lagrangian of the profit maximization problem can be written as

$$\mathcal{L}_m = e_m \left( \pi^1 - w_m^1 \right) + (1 - e_m) \left( \pi^0 - w_m^0 \right)$$

$$+ \mu_1 \left[ e_m u \left( w_m^1 \right) + (1 - e_m) u \left( w_m^0 \right) - \frac{1}{2} e_m^2 \right]$$

$$+ \mu_2 \left[ u \left( w_m^1 \right) - u \left( w_m^0 \right) - e_m \right].$$
that

\[ \frac{\partial L_m}{\partial w^1_m} = \pi^1 - \pi^0 - (w^1_m - w^0_m) - \mu_2 = 0, \]

(D.1)

\[ \frac{\partial L_m}{\partial w^1_m} = -e_m + \left( \mu_1 e_m + \mu_2 + \mu_3 + \mu_4 \right) u'(w^1_m) + \mu_6 = 0, \]

(D.2)

\[ \frac{\partial L_m}{\partial w^0_m} = -(1 - e_m) + [\mu_4 (1 - e_m) - \mu_2 + \mu_5 - \mu_3] u'(w^0_m) + \mu_7 = 0, \]

(D.3)

\[ \frac{\partial L_m}{\partial w^0_m} = \left( -\mu_4 - \frac{1}{2} \mu_5 \right) u'(w^0_m) + \mu_8 = 0. \]

(D.4)

The IC\(_m\) constraint implies that \(w^1_m > w^0_m\) for all \(e_m > 0\) (therefore \(\mu_3 = \mu_6 = 0\)) and the IR constraint is non-binding (\(\mu_1 = 0\)).

Suppose that \(\mu_4 > 0\), we have \(w^1_m = w^0_m\). Substituting this equation into TR\(_3\) yields \(u(w^0_m) \geq u(w^1_m)\), which contradicts IC\(_m\). Therefore, we can conclude that \(\mu_4 = 0\). Suppose that \(\mu_7 > 0\) \((w^0_m = 0)\), TR\(_3\) implies that \(u(w^1_m) = u(w^0_m) = 0\), which contradicts the IC\(_m\) constraint. Thus, we have \(\mu_7 = 0\).

Equation D.3 can be rewritten as \(\mu_5 u'(w^1_m) = (1 - e_m) + \mu_2 u'(w^0_m)\), which implies \(\mu_5 > 0\). Furthermore, if \(w^0_m > 0\) \((\mu_8 = 0)\), then D.4 implies \(\frac{\partial L_m}{\partial w^0_m} < 0\). So, \(w^0_m = 0\) and, therefore, \(u(w^0_m) = 1/2 u(w^1_m)\) from IR\(_3\).

Now, the first-order conditions can be rewritten as

\[ \pi^1 - \pi^0 - w^1_m + w^0_m - \mu_2 = 0, \]

(D.1')

\[ -e_m + \left( \mu_2 - \frac{1}{2} \mu_5 \right) u'(w^1_m) = 0, \]

(D.2')

\[ -(1 - e_m) + (\mu_5 - \mu_2) u'(w^0_m) = 0, \]

(D.3')

The IC\(_m\) constraint can be rewritten as

\[ e_m = \frac{1}{2} u(w^1_m) = u(w^0_m). \]

(IC\(_m'\))
From equation D.1', D.2', D.3' and IC_m', we have

\[ w_1^1 + \frac{u'(w_1^1)}{u'(w_1^0)} w_1^0 - w_1^0 = \pi^1 - \pi^0. \]  

(D.5')

The left hand side of D.5' can be rewritten as

\[ M(w_1^0) = \left( \frac{2 - \rho}{1 - \rho} \right) \left( \frac{1}{2} + 1 \right) w_1^0 + \left( w_1^0 \right)^\rho. \]

Also, \( M(w_1^0) \) is increasing in \( w_1^0 \):

\[ M_w = \frac{\partial M(w_1^0)}{\partial w_1^0} = \left( \frac{2 - \rho}{1 - \rho} \right) \left( \frac{1}{2} + 1 \right) + \rho \left( w_1^0 \right)^{\rho-1} > 0. \]

Denote \( \pi^1 - \pi^0 \) as \( \Delta \pi \). By totally differentiating the equation \( M(v) = \Delta \pi \), we have \( \partial w_1^0 / \partial \Delta \pi = 1 / M_w > 0 \) and \( \partial w_1^1 / \partial \Delta \pi = 2(1/1-\rho)/M_w > 0. \)

E. Proof of Proposition 1

From Lemma 1, 2 and 3, we have

\[ \Pi^{2T} = 2\bar{\theta}y^L + 2\alpha^2 \left[ w_H^H(0) \right]^{2-\rho} \left( 1 - \rho \right)^2, \]

\[ \Pi^{3T} = u\left( w_1^0 \right) \left( \Delta \pi - w_1^1 + w_1^0 \right) + \pi^0 - w_1^0. \]

Since, by envelope theorem, \( \partial B / \partial \theta \) can be written as

\[ \frac{\partial B}{\partial \theta} = \frac{\partial \Pi^{2T}}{\partial \theta} - \frac{\partial \pi^0}{\partial \theta} - u\left( w_1^0 \right) \frac{\partial \Delta \pi}{\partial \theta}, \]

we have

\[ \frac{\partial \Pi^{2T}}{\partial \theta} - \frac{\partial \pi^0}{\partial \theta} = 2 \left[ \alpha^2 - (\alpha \beta)^2 \right] u\left( w_H^H(0) \right) \left( y^H - y^L \right) > 0, \]

and

\[ \frac{\partial \Delta \pi}{\partial \theta} = 2(\alpha \beta)^2 \left[ \frac{1}{2} u\left( w_H^H(\varepsilon) \right) + \frac{1}{2} u\left( w_H^H(-\varepsilon) \right) - u\left( w_H^H(0) \right) \right] < 0. \]

As a result, we can conclude that \( \partial B / \partial \theta > 0. \)
Define $\bar{\beta}$ such that

$$u \left( w_\emptyset^\emptyset \right) \pi^1 + \left[ 1 - u \left( w_\emptyset^\emptyset \right) \right] \pi^\emptyset - \Pi^{2T} = 0.$$  

Since $\partial w_\emptyset^\emptyset / \partial \Delta \pi > 0$, we know that $\partial w_\emptyset^\emptyset / \partial \Delta \pi > 0$ and $\partial \Delta \pi / \partial \beta > 0$, $u( w_\emptyset^\emptyset ) \pi^1 + [ 1 - u( w_\emptyset^\emptyset )] \pi^\emptyset$ is increasing in $\beta$. Therefore, for all $\beta > \bar{\beta}$, we have

$$\Pi^{2T} - u \left( w_\emptyset^\emptyset \right) \pi^1 - \left[ 1 - u \left( w_\emptyset^\emptyset \right) \right] \pi^\emptyset < 0.$$  

As a result,

$$\frac{\partial \Pi^{2T}}{\partial \alpha} - \frac{\partial \pi^\emptyset}{\partial \alpha} = 2 \left( 2\alpha - 2\alpha\beta^2 \right) \frac{\left[ w_d^H (0) \right]^{2-\rho} \left( 1 - \rho \right)^2}{(1 - \rho)^2} > 0,$$

$$\frac{\partial \Delta \pi}{\partial \alpha} = 2 \left( 2\alpha\beta^2 \right) \left[ \frac{w_d^H (\varepsilon)}{2(1 - \rho)^2} + \frac{w_d^H (-\varepsilon)}{2(1 - \rho)^2} \right] > 0.$$  

Finally,

$$\frac{\partial B}{\partial \alpha} = \frac{2}{\alpha} \left\{ \Pi^{2T} - u \left( w_\emptyset^\emptyset \right) \pi^1 - \left[ 1 - u \left( w_\emptyset^\emptyset \right) \right] \pi^\emptyset \right\},$$  

which is negative if $\beta > \bar{\beta}$.

Since the result of $\partial B / \partial \alpha$ critically relies on whether $\beta$ is larger than $\bar{\beta}$, we need to show that $\bar{\beta}$ is bounded between 0 and 1 to make sure the desired situation ($\beta > \bar{\beta}$) exists in our setting. We also show that as $\alpha$ is smaller, the condition $\beta > \bar{\beta}$ is more likely to hold. Specifically, we show that $\partial \bar{\beta} / \partial \alpha > 0$.

By the definition of $\bar{\beta}$ we have

$$\bar{\beta} = \min \left\{ \frac{1}{e_m \left[ \frac{1}{2} \left( \frac{e}{\theta} \right)^{2-\rho} + \frac{1}{2} \left( 1 - \frac{e}{\theta} \right)^{2-\rho} \right] + \left( 1 - e_m \right)} \right\}.$$  

Since $\varepsilon / \theta \in (0, 1)$ and the assumption of interior solution implies that $e_m \in (0, 1)$, it is easy to see that $\bar{\beta} > 0$. Note that $\bar{\beta} < 1$ if and only if

$$\frac{1}{2} \left( 1 + \frac{e}{\theta} \right)^{2-\rho} + \frac{1}{2} \left( 1 - \frac{e}{\theta} \right)^{2-\rho} > 1.$$
Let \( J(x) = \frac{1}{2}(1 + x)^{2-\rho} + \frac{1}{2}(1 - x)^{2-\rho} \), we can see that \( J(\cdot) \) is increasing in \( x \), \( \forall x \in (0, 1) \). Specifically,

\[
\frac{dJ}{dx} = \frac{2 - \rho}{2} \left[ (1 + x)^{1-\rho} - (1 - x)^{1-\rho} \right] > 0,
\]

for all \( x \in (0, 1) \). Together with the fact \( \lim_{x \to 0} J(x) = 1 \), we can conclude that \( J(x) > 1 \) for all \( x \in (0, 1) \). Therefore, we have \( \bar{\beta} < 1 \).

Now we turn to show that \( \partial \bar{\beta} / \partial \alpha > 0 \). From the definition of \( w_m^\emptyset \) in Lemma 3, we have

\[
\frac{\partial w_m^\emptyset}{\partial \alpha} = \frac{\partial w_m^\emptyset}{\partial \beta} = \frac{4\alpha \beta}{(1-\rho)^2} \left[ \frac{1}{2} \left( w_m^H(\varepsilon) \right)^{2-\rho} + \frac{1}{2} \left( w_m^H(-\varepsilon) \right)^{2-\rho} - \frac{1}{2} \left( w_m^H(0) \right)^{2-\rho} \right] > 0.
\]

By the definition of \( \bar{\beta} \) we have

\[
\frac{\partial \bar{\beta}}{\partial \alpha} = -\frac{\left[ 1 - J(\bar{\xi}) \right] \frac{\partial w_m^\emptyset}{\partial \alpha}}{2 \bar{\beta} \left[ e_m J(\bar{\xi}) + (1 - e_m) \right] \left( w_m^H \right)^\rho - \left[ 1 - J(\bar{\xi}) \right] \frac{\partial w_m^\emptyset}{\partial \rho} > 0,
\]

where \( J(\varepsilon/\theta) > 1 \).

References


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本文旨在研究廠商如何調整其內部組織結構與誘因契約，以因應市場環境變化。它說明如果中級經理人的主要任務是收集市場資訊時，市場競爭程度的大小，如何影響廠商內部的層級結構及員工的契約內容。我們提出兩種可能的市場競爭程度的指標。在第一種指標下，增加競爭使得廠商進行組織扁平化，並提供管理者較強的工作誘因。這和近來的實證文獻結果相符。但若以另一種指標衡量競爭時，則會得到相反的結果。因此，本文不僅提供實證文獻所得到關於市場競爭與廠商內部結構的關係一個理論詮釋，並說明這些實證發現在理論上的真正意義。

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