Patent Licensing and Double Marginalization in Vertically-related Markets

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This paper develops a three-stage model in which the input price is expressed by a combination of its monopoly input price and marginal cost. The focus of this paper is on the role of the upstream firm in terms of the degree of its monopoly power in the choice of the outsider patentee’s optimal licensing contract, as the outsider patentee licenses its innovation to the upstream firm. The paper shows that the outsider patentee prefers royalty (fixed-fee) licensing to fixed-fee (royalty) licensing when the degree of the upstream firm’s monopoly power is small (large) regardless of the innovation size. This paper shows that a rise in the degree of double-marginalization may improve the social welfare through the outsider patentee’s switching from royalty licensing to fixed-fee licensing. It also proves that social welfare remains unchanged by the elimination of double-marginalization when the innovation size is large. Finally, the paper is extended by taking into account a two-part tariff.

Keywords: vertically-related markets, double marginalization, outsider patentee, fixed-fee and royalty licensing, two-part tariff

JEL classification: D43, D45, L13

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1 Introduction

This paper aims to explore the following two issues by taking into account vertically-related markets with an outsider patentee, where the cost-reducing innovation is licensed to the upstream firm and the input price is expressed by a combination of its monopoly input price and marginal cost. First of all, what is the outsider patentee’s optimal licensing contract? Secondly, does a decrease in the degree (the elimination) of double-marginalization unambiguously increase social welfare?

Patent licensing has become increasingly popular in past decades. This can be evidenced by the data from Nadiri (1993) indicating that for Japan and the U.K. the total transactions in transnational licensing between the 1970’s and the late 1980’s increased by about 400 percent, for France and the U.S. by about 550 percent, and for West Germany by over 1,000 percent. Moreover, Rostoker (1984) finds from empirical figures that royalties alone account for 39 percent, fixed fees alone for 13 percent, and royalties plus fixed fees for 46 percent of licensing contracts. This demonstrates that fixed-fee and royalty licensing and twpart tariffs are often observed in the real world. Therefore, patent licensing is very important in reality, giving rise to the interest of economists in studying the means by which the patentee would like to license out its innovation.

There is a vast literature discussing the optimal licensing contract, where the innovation is licensed to the final product producers. For example, Kamien and Tauman (1986), Kamien et al. (1992), Muto (1993), Poddar and Sinha (2004), and Kabiraj (2004) focus on the outsider patentee, while Wang (1998), Fauli-Oller and Sandonis (2002); Fauli-Oller and Sandonis (2003), Kabiraj and Marjit (2003), Poddar and Sinha (2004), Arya and Mittendorf (2006), Mukherjee and Pennings (2006), Poddar and Sinha (2010), and Sinha (2010) are directed toward the insider patentee. However, few studies address the issues of patent licensing in relation to the upstream producers, although this kind of licensing is commonplace in the real world. Mukherjee and Ray (2007) is an exception, which shows that the upstream monopolist may license its innovation to the potential entrant if the licensing can increase the number of firms in the downstream market. To the best of our knowledge, there are currently no studies that explore the optimal licensing contract, where the innovation is licensed to the upstream producers. This paper aims to fill this gap in the literature.

In this paper, we express the equilibrium input price as a combination
of its monopoly input price and marginal cost. It can be shown from this combination that as the degree of the upstream firm’s monopoly power becomes smaller (larger), the input price moves closer to its marginal cost (the monopoly input price). It follows that the degree of double-marginalization in the vertically-related markets can be positively linked with the degree of the upstream firm’s monopoly power. The greater the degree of the upstream firm’s monopoly power is, the greater will be the degree of double-marginalization. This enables us to examine whether or not a decrease in the degree (the elimination) of double-marginalization unambiguously improves social welfare.

The results derived in early theoretical works, such as Kamien and Tauman (1986) and Kamien et al. (1992), indicate that as the patent holder stands outside the industry, a fixed-fee licensing arrangement is superior to royalty licensing as firms engage in a Cournot quantity competition with a homogeneous good. Moreover, it is well recognized according to the conventional wisdom that a rise in the degree of double-marginalization unambiguously worsens social welfare. By contrast, the main contributions of this paper are as follows. First of all, the outsider patentee prefers royalty (fixed-fee) licensing to fixed-fee (royalty) licensing, as the degree of the upstream firm’s monopoly power is small (large) regardless of the innovation size. Secondly, a rise in the degree of double-marginalization may improve social welfare through the outsider patentee’s switching from royalty to fixed-fee licensing, as the degree of the upstream firm’s monopoly power is large. Lastly, we show that the level of social welfare remains unchanged by the elimination of double-marginalization when the innovation size is large.

The remainder of this paper is organized as follows. Section 2 constructs a basic model to solve for a general input price and firms’ profits. Section 3 examines the optimal licensing contract in terms of fixed-fee and royalty licensing. Section 4 analyzes the effect of patent licensing on social welfare. Section 5 extends the analysis to the case where the outsider patentee licenses out its innovation by means of a two-part tariff. Section 6 concludes the paper.

2 The Basic Model

Consider a framework consisting of one outsider patentee, one upstream firm, firm 1, and one downstream firm, firm 2. Assume that one unit of final product employs one unit of input. Since both the upstream and the
downstream firms are monopolists, the equilibrium input price, $w$, is thus
determined by the relative strength of the monopoly power between these
two firms. This input price approaches the monopoly price as the monopoly power of the upstream firm is relatively strong, while it approaches the marginal cost, otherwise. As a result, the equilibrium input price can be expressed as a linear combination of the monopoly price and marginal cost, whose weights depend upon the relative strength of the monopoly power between the two firms. Assume further that the upstream firm's marginal cost is a constant $c$, while the downstream firm's marginal cost is the cost of employing the input, $w$. The outsider patentee owns an innovation, which can reduce the upstream firm's marginal production cost by the amount, $\varepsilon$, where $0 < \varepsilon \leq c$, and licenses its innovation to the upstream firm by either a fixed-fee or a pure royalty licensing contract. Assume that the inverse demand function for the final product is a linear function denoted by $p = a - q_2$, where $a$ is the constant reservation price, $q_2$ is the quantity demanded of the final product and $p$ is the market price.

Consider a general model, which can deal with the cases of the absence of licensing, fixed-fee, and royalty licensing simultaneously. The game in question is a three-stage game. In the first stage, the outsider patentee selects an optimal contract and the optimal fixed-fee under fixed-fee licensing or the optimal royalty rate under royalty licensing. In the second stage, the input price is determined by the relative strength of the monopoly power between the upstream and the downstream firms. In the final stage, given the type of licensing contract and the input price, the downstream firm determines its monopoly output. We can derive the sub-game perfect Nash equilibrium by backward induction, beginning with the final stage.

In the final stage, the downstream firm's profit function is defined as:

$$\pi^i_2 = (p - w^i) q^i_2,$$

where $\pi^i_2$ denotes the downstream firm's profit level in case $i$; and the super-script $i = \{N, F, R\}$ is associated with variables in the case of the absence of licensing, fixed-fee, and royalty licensing, respectively.

Substituting the inverse demand function into (1), and then differentiating (1) with respect to $q^i_2$ and letting it equal zero, we can solve for the downstream firm's equilibrium output as follows:

$$q^i_2 = (a - w^i) / 2.$$
Substituting (2) into (1), the downstream firm’s profit function can be rewritten as:

\[ \pi^i_2 = (a - w^i)^2 / 4. \]  

(3)

Next, the upstream firm’s profit function is defined as

\[ \pi^i_1 = (w^i - c^i)q^i_1, \]

where \( q^i_1 \) is the upstream firm’s output in case \( i \). Since we assume that one unit of final product employs one unit of input, the upstream firm’s equilibrium output will be identical to that of the downstream firm, i.e., \( q^i_1 = q^i_2 \).

Thus, the upstream firm’s operating profit function in case \( i \) can be rewritten as follows:

\[ \pi^i_1 = (w^i - c^i) \left( a - w^i \right) / 2, \]  

(4)

where \( \pi^i_1 \) denotes the upstream firm’s operating profit in case \( i \), which is the profit level net of fixed cost.

In the second stage, the upstream firm determines an input price in accordance with the degree of its monopoly power. We analyze the following two polar cases. Firstly, if the upstream firm has full monopoly power, the optimal input price is determined by the upstream firm’s profit-maximizing condition as:

\[ \frac{\partial \pi^i_1}{\partial w^i} = (a - 2w^i - c^i) / 2 = 0. \]

Solving this condition, we can obtain the monopoly input price as

\[ w^i = (a - c^i) / 2 \equiv m^i. \]

Secondly, if the upstream firm has zero monopoly power, the downstream firm can wrest the entire profit of the upstream firm by charging as low an input price as possible until it equals the upstream firm’s marginal cost, i.e., \( w^i = c^i \).

Based on the results derived in the above two polar cases, we can express a general input price as the following form

\[ w^i \equiv \beta m^i + (1 - \beta)c^i, \]

where \( \beta \) denotes the degree of the upstream firm’s monopoly power. The general input price indicates that the equilibrium input price is the monopoly input price as \( \beta = 1 \), equals the marginal cost as \( \beta = 0 \), and lies in between \([c^i, m^i]\) as \( 0 < \beta < 1 \). Thus, the general input price can be expressed as:

\[ w^i = \left[ \beta (a - 3c^i) + 2c^i \right] / 2. \]

(5)

Substituting (5) into (2), we obtain the equilibrium output as follows:

\[ q^i_1 = q^i_2 = (2 - \beta) \left( a - c^i \right) / 4. \]  

(6)

Substituting (5) into (3) and (4), we can derive the reduced profit functions for the upstream and downstream firms as follows:

\[ \pi^i_1 = \beta (2 - \beta) \left( a - c^i \right)^2 / 8, \quad \text{and} \quad \pi^i_2 = \left[ (2 - \beta) \left( a - c^i \right) \right]^2 / 16. \]  

(7)
For the case where patent licensing is absent, by substituting the marginal production cost, \( c^N = c \), into (7), we have the following profit functions:

\[
\pi_1^N = \beta(2 - \beta)(a - c)^2/8, \quad \text{and} \quad \pi_2^N = (2 - \beta)(a - c)^2/16. \quad (8)
\]

3 The Optimal Licensing Contract

In this section, we examine the optimal licensing contract in stage 1. In what follows we first explore the optimal fixed-fee under fixed-fee licensing, and then the optimal royalty rate under royalty licensing. Finally, we compare the patentee’s profit between fixed-fee and royalty licensing to work out the optimal licensing contract.

3.1 Fixed-fee licensing

In this subsection, we analyze the optimal fixed-fee in the case where the outsider patentee licenses a cost-reducing innovation to the upstream firm by means of fixed-fee licensing. Following the related literature, the outsider patentee can extract the entire extra benefits caused by the licensing via charging a fixed-fee. Thus, the optimal fixed-fee can be defined as the difference in profit for the licensee between accepting and rejecting the license, i.e., \( \pi_1^F - F = \pi_1^N \), where the variable \( F \) denotes the amount of the fixed-fee.

Substituting the marginal production cost under fixed-fee licensing, \( c^F = c - \varepsilon \), into (6) and (7), the output and the profit for the upstream firm and the downstream firm can be derived as follows:

\[
q_1^F = q_2^F = (2 - \beta)(a - c + \varepsilon)/4, \quad (9)
\]

\[
\pi_1^F - F = \beta(2 - \beta)(a - c + \varepsilon)^2/8 - F, \quad \text{and} \quad \pi_2^F = (2 - \beta)(a - c + \varepsilon)^2/16. \quad (10)
\]

The optimal fixed-fee, i.e., the outsider patentee’s profit under fixed-fee licensing, can be derived by (8) and (10) as follows:

\[
F = \pi_1^F - \pi_1^N = \beta(2 - \beta)[(a - c + \varepsilon)^2 - (a - c)^2]/8. \quad (11)
\]

Recall that \( 0 \leq \beta \leq 1 \). Equation (11) shows that a rise in the degree of the upstream firm’s monopoly power increases the optimal fixed-fee. This result emerges because the outsider patentee can wrest the whole extra benefits
caused by licensing the innovation to the upstream firm, and the greater the upstream firm’s monopoly power is, the higher will be the upstream firm’s profit. Thus, we can establish the following lemma:

**Lemma 1.** Suppose that the outsider patentee licenses its innovation to the upstream firm by fixed-fee licensing. A rise in the degree of the upstream firm’s monopoly power increases the amount of the optimal fixed-fee.

### 3.2 Royalty Licensing

We turn to studying the optimal royalty rate in the case where the outsider patentee licenses its innovation by means of royalty licensing.

Substituting the marginal production cost under royalty licensing, $c^R = c - \varepsilon + r$, where $r$ denotes the royalty rate, into (6) and (7), we can derive the equilibrium output and the equilibrium profit for the upstream firm and the downstream firm as follows:

$$q_1^R = q_2^R = \frac{(2 - \beta)(a - c + \varepsilon - r)}{4}, \quad (12)$$

$$\pi_1^R = \beta(2 - \beta)(a - c + \varepsilon - r)^2/8,$$

and

$$\pi_2^R = [(2 - \beta)(a - c + \varepsilon - r)]^2/16. \quad (13)$$

In the first stage, the outsider patentee determines the optimal royalty rate to maximize its profit. It should be noted that the optimal royalty rate has to be no greater than the innovation size, i.e., $r \leq \varepsilon$, to ensure that the upstream firm would like to accept the licensing. Thus, the outsider patentee’s problem can be described as follows:

$$\max_r R = rq_1^R = r[(2 - \beta)(a - c + \varepsilon - r)]/4, \quad \text{subject to } r \leq \varepsilon. \quad (14)$$

where $R$ denotes the outsider patentee’s royalty revenue under royalty licensing.

Differentiating (14) with respect to $r$, we obtain:

$$\frac{\partial R}{\partial r} = (2 - \beta)(a - c + \varepsilon - 2r)/4 \geq 0. \quad (15)$$

By solving (15), we derive the optimal royalty rate as follows:

$$r = \begin{cases} 
\varepsilon, & \text{if } \varepsilon < a - c, \\
(a - c + \varepsilon)/2, & \text{if } \varepsilon \geq a - c.
\end{cases} \quad (16)$$
Equation (16) shows that the optimal royalty rate equals the innovation size when the innovation size is small, say, $\varepsilon < a - c$, while it is an interior solution equaling $[(a - c + \varepsilon)/2]$, otherwise. Thus, the optimal royalty rate has nothing to do with the degree of the upstream firm's monopoly power. Intuitively, we can regard the outsider patentee as a monopolist that charges the upstream firm a profit-maximizing royalty rate. When the innovation size is so small that the innovation size is smaller than the monopoly royalty rate, the outsider patentee will charge a royalty rate equaling the innovation size because this is the highest royalty rate that the licensee is willing to pay. When the innovation size is so large that the innovation size is larger than the monopoly royalty rate, the outsider patentee will charge the monopoly royalty rate equaling $[(a - c + \varepsilon)/2]$ to maximize its profit. Since the monopoly royalty rate is determined by the condition that the marginal revenue equals marginal cost, which is assumed to be zero, it will be located at the center of the linear derived demand curve. It is worth pointing out that the price elasticity of demand at the center of the linear derived demand curve is unit-elastic. It follows that the optimal royalty rate at the center of the linear demand curve equals $[(a - c + \varepsilon)/2]$ regardless of the degree of the upstream firm’s monopoly power. Thus, we can conclude that the optimal royalty rate has nothing to do with the degree of the upstream firm’s monopoly power regardless of the value of the innovation size.

Substituting (16) into (12) and (13), we obtain the equilibrium output and profit of the upstream firm and the downstream firm under royalty licensing as follows:

$$q^R_1 = q^R_2 = \begin{cases} (2 - \beta) (a - c)/4, & \text{if } \varepsilon < a - c, \\ (2 - \beta) (a - c + \varepsilon)/8, & \text{if } \varepsilon \geq a - c. \end{cases}$$

(17)

$$\pi^R_1 = \begin{cases} \beta (2 - \beta) (a - c)^2/8, & \text{if } \varepsilon < a - c, \\ \beta (2 - \beta) (a - c + \varepsilon)^2/32, & \text{if } \varepsilon \geq a - c. \end{cases}$$

(18)

$$\pi^R_2 = \begin{cases} (2 - \beta)^2 (a - c)^2/16, & \text{if } \varepsilon < a - c, \\ (2 - \beta)^2 (a - c + \varepsilon)^2/64, & \text{if } \varepsilon \geq a - c. \end{cases}$$

(19)

Next, substituting (16) into (14) gives the outsider patentee’s profit (royalty revenue) under royalty licensing as follows:

$$R = \begin{cases} \varepsilon (a - c) (2 - \beta)/4, & \text{if } \varepsilon < a - c, \\ (a - c + \varepsilon)^2 (2 - \beta)/16, & \text{if } \varepsilon \geq a - c. \end{cases}$$

(20)

We find from (20) that a rise in the degree of the upstream firm’s monopoly power decreases the outsider patentee’s profit under royalty licensing. The
intuition can be stated as follows. First of all, we have shown in (16) that the optimal royalty rate has nothing to do with the degree of the upstream firm’s monopoly power. The intuition underlying this result has been stated previously. Secondly, a rise in the upstream firm’s monopoly power enhances the upstream firm’s ability to capture more profit by raising the input price. This will decrease the output of the downstream firm due to the assumption that one unit of the final product employs one unit of input. Based on the above analysis, a rise in the degree of the upstream firm’s monopoly power decreases the output of the downstream firm but has no impact on the royalty rate. Thus, the royalty revenue and then the outsider patentee’s profit will decrease. Accordingly, we have the following lemma:

**Lemma 2.** Suppose that the outsider patentee licenses its innovation to the upstream firm by royalty licensing. A rise in the degree of the upstream firm’s monopoly power decreases the outsider patentee’s profit.

### 3.3 The Optimal Licensing Contract

We are now in a position to examine the optimal licensing contract. This can be done by deducting (11) from (20). However, we find from (20) that there are two possible solutions for the outsider patentee’s profit, depending upon the magnitude of the innovation size. In what follows, we discuss two cases, where the innovation size is small and large, respectively.

For the case where the innovation size is small, say, \(\varepsilon < a - c\):

In this case, the optimal royalty rate equals the innovation size. Deducting (11) from (20) gives:

\[
R - F \geq (\leq) 0, \quad \text{if } \beta \leq (>) (2a - c)/(2a - 2c + \varepsilon) \equiv \beta_A. \tag{21}
\]

Equation (21) shows that the outsider patentee prefers fixed-fee (royalty) licensing, as the degree of the upstream firm’s monopoly power is large (small), say, larger (smaller) than \(\beta_A\). The intuition behind this result is as follows. As the degree of the upstream firm’s monopoly power is large, the upstream firm has the power to extract a larger profit from the downstream firm. This will attract the outsider patentee to choose fixed-fee licensing to enhance the production efficiency of the upstream firm by reducing its marginal production cost. Then, the outsider patentee can earn a larger profit by wresting
the entire extra benefits of the upstream firm from accepting the license. On
the contrary, as the degree of the upstream firm’s monopoly power is small,
the downstream firm has the ability to extract a larger profit from the up-
stream firm, leading to a lower input price and a larger output of the final
product. This will attract the outsider patentee to choose royalty licensing
to earn more royalty revenue. Moreover, recall Lemmas 1 and 2 whereby
the outsider patentee’s profit under royalty (fixed-fee) licensing is decreasing
(increasing) in the degree of the upstream firm’s monopoly power. As a re-
result, the outsider patentee will choose fixed-fee licensing when the degree of
the upstream firm’s monopoly power is large, whereas it will select royalty
licensing otherwise.

For the case where the innovation size is large, say, \( \varepsilon \geq a - c \):

In this case, the optimal royalty rate is lower than the innovation size. De-
ducting (11) from (20), we obtain:

\[
R - F \geq \begin{cases} < 0, & \text{if } \beta \leq \hat{\beta}_B, \\
& \text{or } \beta > \hat{\beta}_B \end{cases}
\]

where

\[
\hat{\beta}_B = \frac{[(a - c)^2 + \varepsilon(2a - 2c + \varepsilon)]}{2\varepsilon(2a - 2c + \varepsilon)}.
\]  

We find from (22) that the outsider patentee prefers fixed-fee (royalty) li-
censing, as the degree of the upstream firm’s monopoly power is large (small),
say, larger (smaller) than \( \hat{\beta}_B \). The same intuition as that in the case of small
innovation size carries over to this case.

In sum, we find from (21) and (22) that the outsider patentee prefers
fixed-fee (royalty) licensing when the degree of the upstream firm’s monopoly
power is large (small), regardless of the innovation size. Thus, we can estab-
lish:

**Proposition 1.** Supposing that the outsider patentee licenses its innovation
to the upstream firm, the outsider patentee would like to choose fixed-fee
(royalty) licensing when the degree of the upstream firm’s monopoly power
is large (small), regardless of the innovation size.

Proposition 1 is sharply different from those propositions derived in the
early literature where the outsider patentee always prefers fixed-fee licensing
to royalty licensing in the absence of vertically-related markets.
4 Social Welfare and Double Marginalization

In this section, we study the impact of a rise in the degree of double-marginalization, measured by a rise in the degree of the upstream firm's monopoly power, on social welfare. The social welfare is defined as the sum of the consumer's surplus, \((q_1^i)^2 / 2\), and the firms' aggregate profits consisting of the profits for the downstream firm, the upstream firm and the outsider patentee.

We can calculate the social welfare under fixed-fee licensing by (9) – (11) as follows:

\[
SW^F = (2 - \beta)(6 + \beta)(a - c + \varepsilon)^2 / 32. 
\]

(23)

Similarly, the social welfare under royalty licensing can be derived from (17) – (20) as follows:

\[
SW^R = \begin{cases} 
(2 - \beta)(a - c)[(6 + \beta)(a - c) + 8\varepsilon] / 32, & \text{if } \varepsilon < a - c, \\
(2 - \beta)(14 + \beta)(a - c + \varepsilon)^2 / 128, & \text{if } \varepsilon \geq a - c.
\end{cases} 
\]

(24)

Deducting (24) from (23), we obtain:

\[
SW^F - SW^R = \begin{cases} 
\varepsilon(2 - \beta)[6(a - c)(2 - \beta) + \varepsilon(6 + \beta)] / 32 > 0, & \text{if } \varepsilon < a - c, \\
(a - c + \varepsilon)^2(2 - \beta)(10 + 3\beta) / 1128 > 0, & \text{if } \varepsilon \geq a - c.
\end{cases} 
\]

(25)

Given the degree of the upstream firm's monopoly power, equation (25) shows that the social welfare under fixed-fee licensing is definitely larger than that under royalty licensing. This result arises from the fact that the upstream firm produces at a lower marginal cost, i.e., it is more efficient, under fixed-fee licensing than under royalty licensing.

Next, we can calculate from (23) and (24) that \(\partial SW^F / \partial \beta < 0\) and \(\partial SW^R / \partial \beta < 0\), indicating that the social welfare worsens as the degree of the double-marginalization increases in both cases of royalty and fixed-fee licensing. This result occurs because the input price rises and then the marginal production cost of the downstream product increases, as the degree of double-marginalization rises. This will in turn reduce the output of the downstream good, and then worsen social welfare.

We use Figure 1, in which the horizontal axis denotes the degree of double-marginalization and the vertical axis represents the level of social...
welfare, to illustrate the locus of social welfare for a large innovation size, say, \( \varepsilon \geq a - c \). In Figure 1, the curve \( SW^F(SW^R) \) denotes the locus of social welfare under fixed-fee (royalty) licensing for a large innovation size. Recalling the terms \( \partial SW^F/\partial \beta < 0 \) and \( \partial SW^R/\partial \beta < 0 \), both curves are negatively sloping and concave in \( \beta \). Moreover, we find from (25) that curve \( SW^R \) lies below curve \( SW^F \). Next, we have shown in (22) that the outsider patentee will choose royalty licensing when the degree of the upstream firm's monopoly power is small, say, \( 0 \leq \beta \leq \hat{\beta}_B \), whereas it will select fixed-fee licensing when \( \hat{\beta}_B < \beta \leq 1 \). It follows that the social welfare jumps from curve \( SW^R \) up to curve \( SW^F \) when the degree of the upstream firm's monopoly power is larger than the threshold level \( \hat{\beta}_B \). Moreover, by substituting \( \beta = 1 \) into (23) and \( \beta = \hat{\beta}_B \) into (24), the difference in the level of social welfare between fixed-fee and royalty licensing can be derived as follows:

\[
SW^F(\beta = 1) - SW^R(\beta = \hat{\beta}_B) = (a - c + \varepsilon)^4[(a - c + 5\varepsilon)^2 + 40\varepsilon(a - c)]/ \\
[512\varepsilon^2(2a - 2c + \varepsilon)^2] > 0. \tag{26}
\]

Equation (26) shows that the level of social welfare under fixed-fee licensing at \( \beta = 1 \) is higher than that under royalty licensing at \( \beta = \hat{\beta}_B \). Re-

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1From (23) and (24), we can obtain that \( \partial^2 SW^F/\partial \beta^2 < 0 \) and \( \partial^2 SW^R/\partial \beta^2 < 0 \).
call that \( \partial SW^F / \partial \beta < 0 \). We can easily find that the difference in the level of social welfare under fixed-fee licensing at \( \beta \) and that under royalty licensing at \( \beta = \hat{\beta}_B \), \( SW^F (\beta) - SW^R (\beta_B) \), is positive in the region \( \hat{\beta}_B < \beta \leq 1 \). As a result, a rise in the degree of double-marginalization improves social welfare in this region through the outsider patentee’s switching from royalty licensing to fixed-fee licensing.\(^2\) However, it is worth emphasizing that the conventional wisdom indicates that a rise in the degree of double-marginalization unambiguously worsens social welfare.

Based on the above analysis, we have the following proposition:

**Proposition 2.** Suppose that the outsider patentee licenses its innovation to the upstream firm. A rise in the degree of double-marginalization may improve social welfare through the outsider patentee’s switching from royalty licensing to fixed-fee licensing.

We proceed to examine whether or not social welfare unambiguously improves by the elimination of double-marginalization. Recall that the degree of double-marginalization is zero as \( \beta = 0 \), and equals unity as \( \beta = 1 \). Thus, the effect of the elimination of the double-marginalization on social welfare can be calculated by substituting \( \beta = 1 \) into (23) and \( \beta = 0 \) into (24) as follows:

\[
SW^R (\beta = 0) - SW^F (\beta = 1) =
\begin{cases} 
5(a - c)^2 + 2(a - c)e - 7e^2 \geq 0, & \text{if } e < (a - c), \\
0, & \text{if } e \geq (a - c).
\end{cases}
\]

Equation (27) shows that the level of social welfare remains unchanged by the elimination of double-marginalization as the innovation size is large, say \( [e \geq (a - c)] \), while it increases otherwise. Thus, we can establish:

**Proposition 3.** Suppose that the outsider patentee licenses its innovation to the upstream firm in terms of fixed-fee and royalty licensing. The level of social welfare remains unchanged by the elimination of double-marginalization as the innovation size is large, say \( [e \geq (a - c)] \), while it increases otherwise.

Propositions 2 and 3 are sharply different from the conventional wisdom, in which a decrease in the degree (the elimination) of double-marginalization unambiguously improves social welfare. By contrast, we show that a

\(^2\)The same result can be derived in the case where the innovation size is small, say, \( e < a - c \).
decrease in the degree of double-marginalization may worsen social welfare through the outsider patentee’s switching from fixed-fee licensing to royalty licensing. Moreover, the level of social welfare remains unchanged by the elimination of double-marginalization, as the innovation size is large.

5 Two-Part Tariff

In this section, we extend our analysis to the case where the outsider patentee licenses its innovation to the upstream firm by means of a two-part tariff, i.e., a fixed fee plus a per-unit royalty rate. Note that the licensee’s marginal production cost becomes $c^T = c - \varepsilon + r$, if it accepts the license under the two-part tariff, where the superscript “$T$” denotes the case under the two-part tariff. Furthermore, the profit function for the licensee consists of an extra fixed-fee cost in this case. Following the same procedure, by substituting $c^T = c - \varepsilon + r$ into (6) and (7), we can derive the upstream firm’s operating profit function under the two-part tariff as follows:

$$\pi^T_1 = \beta(2 - \beta) \left( a - c + \varepsilon - r^T \right)^2 / 8. \quad (28)$$

The outsider patentee’s profit consists of royalty revenue and the fixed fee paid by the licensee, which can be expressed as follows:

$$M = r^T q^T_1 + F^T = r^T \left[ (2 - \beta) (a - c + \varepsilon - r^T) \right] / 4 + F^T,$$

where $M$ denotes the outsider patentee’s profit under the two-part tariff; and $F^T$ denotes the fixed fee in the case of licensing by means of the two-part tariff.

In the first stage, the outsider patentee chooses the optimal royalty rate and fixed fee to maximize its profit under the constraint that the licensee would like to accept the license. This can be described as follows:

$$\max_{r^T, F^T} M = r^T \left[ (2 - \beta) (a - c + \varepsilon - r^T) \right] / 4 + F^T,$$

s.t. $\pi^T_1 - F^T \geq \pi^N_1. \quad (29)$

We find from (29) that the outsider patentee’s profit is monotonically increasing in the fixed fee. Thus, the outsider would charge a fixed fee as high as possible until the constraint is binding, i.e., $F^T = \pi^T_1 - \pi^N_1$. By substituting this binding constraint into (29) and then differentiating (29) with
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respect to the royalty rate and letting the differential equation equal zero, we can solve for the optimal royalty rate as follows:

\[
\begin{align*}
\tau^* & = \begin{cases} 
\varepsilon, & \text{if } 0 \leq \beta \leq (a - c - \varepsilon)/(a - c), \\
(1 - \beta)(a - c + \varepsilon)/(2 - \beta), & \text{if } (a - c - \varepsilon)/(a - c) < \beta < 1, \\
0, & \text{if } \beta = 1.
\end{cases}
\end{align*}
\]

(30)

Recall that the optimal fixed-fee requires the constraint to be binding, i.e., \( F^T = \pi_1^T - \pi_1^N \). By substituting (30) into (28), we can derive the optimal fixed-fee from (28) and (8) as follows:

\[
\begin{align*}
F^T & = \begin{cases} 
0, & \text{if } 0 \leq \beta \leq (a - c - \varepsilon)/(a - c), \\
\frac{\beta(2-\beta)}{8} \left[ \frac{(a - c + \varepsilon)^2}{(a - c)^2} - (a - c)^2 \right], & \text{if } (a - c - \varepsilon)/(a - c) < \beta < 1, \\
\beta(2 - \beta) \left[ (a - c + \varepsilon)^2 - (a - c)^2 \right]/8, & \text{if } \beta = 1.
\end{cases}
\end{align*}
\]

(31)

We can find from (30) and (31) that, for a small innovation, say, \( \varepsilon < a - c \), the outsider patentee’s optimal licensing contract is pure royalty licensing when the degree of the upstream firm’s monopoly power (\( \beta \)) is small, say, \( [\beta \leq (a - c - \varepsilon)/(a - c)] \), is a two-part tariff when \( \beta \) is moderate, say, \( [(a - c - \varepsilon)/(a - c) < \beta < 1] \), and is pure fixed-fee licensing when \( \beta \) equals unity. The intuition stated in Proposition 1 applies to the above results.

When the degree of the upstream firm’s monopoly power (\( \beta \)) is small, the downstream firm has the power to extract a larger profit from the upstream firm. This will attract the outsider patentee to choose pure royalty licensing in order to prevent the downstream firm from wresting the licensing benefits. On the contrary, when \( \beta \) is large, the outsider patentee will charge a lower royalty rate to enhance the production efficiency of the upstream firm by reducing its marginal production cost, and meanwhile charge a fixed fee to take away the rest of the licensing benefits. Finally, when \( \beta \) equals unity, the whole of the market profits are totally captured by the upstream firm. The outsider patentee will choose pure fixed-fee licensing to enhance the production efficiency of the upstream firm, and take away the whole of the licensing benefits. Next, for a large innovation, say, \( \varepsilon \geq a - c \), we can figure out that \([(a - c - \varepsilon)/(a - c) < 0 \leq \beta < 1]\). Thus, we find from (30) and (31) that pure royalty licensing can never occur in this case.

Based on the above analysis, we can establish:
Proposition 4. Suppose that the outsider patentee licenses its technology to an upstream monopolist by means of a two-part tariff. For the case of a small innovation, say, \( \varepsilon < a - c \), the outsider patentee’s optimal licensing contract is pure royalty licensing when the degree of the upstream firm’s monopoly power (\( \beta \)) is small, say, \( \beta \leq (a - c - \varepsilon)/(a - c) \), is a two-part tariff when \( \beta \) is moderate, say, \( (a - c - \varepsilon)/(a - c) < \beta < 1 \), and is pure fixed-fee licensing when \( \beta \) equals unity. However, for the case of a large innovation, say, \( \varepsilon \geq a - c \), a pure royalty licensing can never occur.

We turn to examine the impact of a rise in the degree of double-marginalization on social welfare. Recall that the social welfare is measured as the sum of the consumer’s surplus, \( (q_i^1)^2/2 \), and firms’ aggregate profits consisting of the profits of the downstream firm, the upstream firm and the outsider patentee. Thus, the social welfare under the two-part tariff can be derived as follows:

\[
SW^T = \begin{cases} 
(2 - \beta)(a - c)[(6 + \beta)(a - c) + 8\varepsilon]/32, & \text{if } 0 \leq \beta \leq (a - c - \varepsilon)/(a - c), \\
7(a - c + \varepsilon)^2/32, & \text{if } (a - c - \varepsilon)/(a - c) < \beta \leq 1.
\end{cases}
\]  
(32)

Differentiating (32) with respect to \( \beta \), we obtain:

\[
\frac{dSW^T}{d\beta} = \begin{cases} 
-(a - c)[(a - c)(2 + \beta) + 4\varepsilon]/16 < 0, & \text{if } 0 \leq \beta \leq (a - c - \varepsilon)/(a - c), \\
0, & \text{if } (a - c - \varepsilon)/(a - c) < \beta \leq 1.
\end{cases}
\]  
(33)

Equation (33) shows that for a small innovation, say, \( \varepsilon < a - c \), social welfare worsens due to a rise in the degree of double-marginalization when this degree is small, say, \( \beta \leq (a - c - \varepsilon)/(a - c) \), while it remains unchanged otherwise. Moreover, we also find from (33) that the elimination of double-marginalization is definitely welfare-improving. We use Figure 2 to illustrate the locus of social welfare under the two-part tariff. In Figure 2, \( SW^T \), \( SW^F \) and \( SW^R \) represent the locus of social welfare under the two-part tariff, pure fixed-fee and pure royalty licensing, respectively. Recall that

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3. The calculation is available from the authors upon request.

4. Equation (33) shows that social welfare is monotonically decreasing in the degree of double-marginalization when \( \beta \leq (a - c - \varepsilon)/(a - c) \), while it remains unchanged otherwise. It follows that \( SW^T (\beta = 0) > SW^T (\beta = 1) \).
the outsider patentee’s optimal licensing contract is pure royalty licensing when the degree of the upstream firm’s monopoly power ($\beta$) is small, say, $[\beta \leq (a - c - \varepsilon)/(a - c)]$, is a two-part tariff when $\beta$ is moderate, say, $[(a - c - \varepsilon)/(a - c) < \beta < 1]$, and is pure fixed-fee licensing when $\beta$ equals unity. Thus, the locus of social welfare lies on curve $SW^R$ when $[\beta \leq (a - c - \varepsilon)/(a - c)]$, on curve $SW^T$ when $[(a - c - \varepsilon)/(a - c) < \beta < 1]$, and on curve $SW^F$ when $\beta$ equals unity. Note that both the two-part tariff and pure fixed-fee licensing can improve social welfare by decreasing the licensee’s production cost, while pure royalty licensing has no impact on the level of social welfare because the licensee’s production cost remains unchanged. Similar to the intuition stated in Section 4, we can summarize by stating that a rise in the degree of double-marginalization may have two effects. First, it definitely worsens social welfare by increasing the input price. This effect is exactly the conventional effect. Second, it may improve social welfare by decreasing the licensee’s marginal production cost through the
two-part tariff. In this paper, we find that the second effect covers the first effect so that the level of social welfare remains unchanged under licensing by means of the two-part tariff when the degree of double-marginalization is large. On the contrary, when the degree of double-marginalization is small, the outsider patentee will choose to license by means of the pure royalty rate. Thus, the second effect vanishes, resulting in a decrease in the level of social welfare. Moreover, since a rise in the degree of double-marginalization always decreases the level of social welfare in the region \(0 \leq \beta < 1\), it follows that the elimination of double-marginalization is welfare-improving.

Next, for a large innovation, say, \(\varepsilon \geq a - c\), we can figure out that \(\frac{(a - c - \varepsilon)}{(a - c)} < 0 \leq \beta \leq 1\). Substituting this inequality into (33), we obtain that \(\partial SW^T/\partial \beta = 0\) holds for \(0 \leq \beta \leq 1\). Thus, the level of social welfare always remains unchanged by a rise in the degree of double-marginalization. Moreover, the level of social welfare also remains unchanged by the elimination of double-marginalization. This result can be illustrated by Figure 3. The same intuition as that in the case of a small innovation applies in this case.

Based on the above analysis, we can establish the following proposition:

![Figure 3: The locus of social welfare for a large innovation in the case of licensing by means of a two-part tariff](image-url)
**Proposition 5.** Suppose that the outsider patentee licenses its technology to an upstream monopolist by means of a two-part tariff. We propose that:

(i) for the case of a small innovation, say, \( \varepsilon < a - c \), the level of social welfare remains unchanged by a rise in the degree of double-marginalization when this degree is large, say, \( \beta \geq (a - c - \varepsilon)/(a - c) \), while it decreases otherwise. Moreover, the elimination of double-marginalization is definitely welfare-improving.

(ii) for the case of a large innovation, say, \( \varepsilon \geq a - c \), both a rise in the degree and the elimination of the degree of double-marginalization keep the level of social welfare unchanged.

The results derived in Proposition 5 are different from those derived in Proposition 2, in which a rise in the degree of double-marginalization may improve social welfare through the outsider patentee’s switching from royalty licensing to fixed-fee licensing.

### 6 Concluding Remarks

This paper has developed a three-stage model, in which the equilibrium input price is expressed by a combination of the monopoly input price and its marginal cost. The focus of this paper is on the role of the upstream firm’s monopoly power in the choice of optimal licensing contract, as the outsider patentee licenses out its innovation to the upstream firm. Several striking results are derived as follows.

First of all, this paper shows that the outsider patentee prefers royalty (fixed-fee) licensing to fixed-fee (royalty) licensing when the degree of the upstream firm’s monopoly power is small (large), regardless of the innovation size. Secondly, it proves that a rise in the degree of double-marginalization may improve social welfare through the outsider patentee’s switching from royalty to fixed-fee licensing. Finally, we show that the level of social welfare remains unchanged by the elimination of double-marginalization when the innovation size is large.

The policy implication that can be drawn from the results derived in this paper is that the government does not need to worry about the upstream firm’s monopoly power in the case where the outsider patentee switches its licensing policy from royalty to fixed-fee licensing due to a rise in the degree.
of double-marginalization. Accordingly, when contrasted with the conventional assertion that the larger the upstream firm that wields the monopoly power is, the more likely it is that the government will regulate the upstream firm, this paper argues that the government should let such regulation go in the presence of outside patent licensing.

References


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垂直相關市場中的技術授權與雙重邊際化

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本文設立一個包括垂直相關市場的三階段賽局模型, 定義中間財價格與其邊際成本的線性組合。本文主要在探討當產業外授權廠商授權給上游廠商時, 上游廠商的獨佔力程度在最適授權策略的決策中所扮演的角色。本文發現當上游廠商的獨佔力相對小 (大) 時, 無論創新程度 (innovation size) 爲何, 最適授權策略為單位權利金授權 (固定權利金授權)。其次, 本文發現提高雙重邊際化程度, 可藉由產業外授權廠商的授權策略由單位權利金授權變成固定權利金授權, 提升社會福利水準。再者, 本文証明當創新程度夠大時, 即使完全消除雙重邊際化, 社會福利水準仍維持不變。最後, 本文也考慮了兩部授權策略下的最適授權策略與其對社會福利的影響。

關鍵詞: 垂直相關市場, 雙重邊際化, 產業外授權廠商, 固定權利金與單位權利金授權, 兩部授權

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