Internationalized Production and Monetary Policy Coordination

Hsiao-Lei Chu and Nan-Kuang Chen*

We investigate the welfare gains of international monetary coordination in the presence of internationalized production - firms employ both domestic and foreign produced intermediate goods. Incorporating internationalized production into a two-country intertemporal general equilibrium model, we study the role for a delegated monetary authority in implementing the competitive allocation. We find that the range of parameter values that supports delegated policy coordination as well as the sizes of welfare gains from policy coordination shrink when the extent of internationalized production rises. Thus, international trade in intermediate goods diminishes the viability of an international monetary authority.

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1 Introduction

In this paper we investigate the efficiency and welfare gains of international monetary cooperation, emphasizing a prevalent feature of international trade - firms employ both domestic and foreign manufactured intermediate goods

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to produce final goods - dubbed as internationalized production.\footnote{1} Incorporating internationalized production into a two-country intertemporal general equilibrium model with nominal rigidity and imperfect competition, we characterize the conditions under which a non-cooperative equilibrium and a decentralized cooperative equilibrium result in competitive allocation, and then study the role for a delegated monetary authority in improving the welfare of both countries.

To establish a case for the need of international policy cooperation, we first look back to Corsetti and Pesenti (2001) results in a two-country general equilibrium model with rigid nominal prices of consumption goods and monopolistic distortions in production. Their approach allows a closed-form solution and is thus able to illustrate the effects of large monetary shocks on welfare. They show that, while the gap between actual output and potential output can be eliminated by a welfare-maximizing expansionary monetary shock in a closed economy as pointed out by Blanchard and Kiyotaki (1987), a welfare-maximizing expansionary monetary shock fails to close the gap in an open economy. The failure to exploit the welfare gains results from a terms-of-trade effect which is associated with openness itself - that is, a positive monetary shock deteriorates domestic agents’ purchasing power. When an expansionary monetary shock reduces the relative price of home goods to foreign goods and raises the demand for home goods, the trade-off between the welfare loss from working more and the welfare gain generated by increased sales in home goods is complicated by the welfare loss from a lower purchasing power for domestic agents. Therefore, in an open economy a monetary authority prefers a less expansionary monetary shock than required to achieve efficient production, which is called contractionary bias.

\footnote{Internationalized production may exist through international trading in intermediate goods, establishing foreign-owned firms, or foreign direct investment, and so on. Much evidence supports that internationalized production is more prevalent. For example, Campa and Goldberg (1997) find that imported inputs as a share of the value of production for selected countries have increased considerably in the last two decades. Hummels et al. (1998) show that, even though vertical trade (imported inputs which are used to manufacture goods for export) accounts for a relatively small fraction of total trade, its contribution to total export growth far exceeds its share of total trade in almost all the sample countries. Jungnickel and Keller (2003) report the scales of sales and employment of foreign-owned firms for selected countries. They find that the sales and employment in percentage of the respective values for total manufacturing rapidly increased during 1985 and 1998, except in Japan and Italy. In particular, foreign-owned firms employed more than a quarter of the total employment in the manufacturing sectors of France and Belgium in 1998.}
Corsetti and Pesenti (2001) thus argue that only if both countries raise monetary stock unexpectedly in a parallel way, so as to eliminate the terms-of-trade effect, can Blanchard and Kiyotaki (1987) result be restored. Corsetti and Pesenti also show that an expansionary monetary shock at home always benefits foreign agents through raising their purchasing power. Thus, a country will not retaliate against a foreign currency devaluation by monetary shocks.

Based on Corsetti and Pesenti (2001) Benigno (2002) investigates the efficiency and credibility of international monetary cooperation. He shows that in a Nash (non-cooperative) equilibrium both countries are operating at a monopolistic allocation because variations in terms-of-trade prevent each country from engaging in a larger monetary expansion to achieve first-best production, while a cooperative agreement on fixed terms of trade can reach the competitive equilibrium allocation. The cooperative agreement, however, can never be credible, because a country can gain from contracting its money stock to improve terms of trade. Due to inefficiency of the Nash equilibrium and fragility of the cooperative agreement, Benigno establishes a rationale for a monetary delegate (and even a common currency). He finds that whether a monetary delegate is viable depends on the substitutability of the traded final goods and the share of each final goods in world consumption.

The primary objective of this paper is to investigate the role of internationalized production in the recent debates over the desirability of international monetary coordination. Our analysis is based on the model by Chu (2005) who introduces a key feature – in each country a fraction of the population produces exported intermediate goods and receives payments in foreign currency – into Corsetti and Pesenti (2001). In contrast to Corsetti

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2 Bacchetta and Wincoop (2005) argue that the market share of an exporting country in a foreign market and the extent to which products of domestic firms can be substitutes for those of competing foreign firms are the two key factors empirically relevant to invoicing choice. Specifically, they show that the lower the exporter's market share is in an industry and the less differentiated the products are, the more likely it is that firms will price in the importer's currency. In this paper intermediate goods are the counterpart of the traded goods in Bacchetta and van Wincoop's paper. Since the ratio of imported to domestic intermediate goods is lower than one in general and the intra-industry substitution of intermediate goods is much higher than inter-industry substitution of intermediate goods, we assume that the price of imported intermediate goods is contracted in terms of the domestic currency. The invoicing choice has also been found by Devereux and Engel (1999b) and others to play a critical role for optimal monetary policy and the choice of exchange rate regime. For related
and Pesenti (2001), Chu (2005) derives the range of structures of internationalized production where an expansionary monetary shock is beggar-thy-neighbor, which also raises the issue of international retaliation and cooperation in monetary policy and thus motivates this paper. The main difference of this paper from Chu (2005) is that we allows for these countries to coordinate their monetary policies. We then discuss the viability and welfare effect of different types of cooperation.

We first characterize the equilibrium outcomes of a non-cooperative equilibrium, i.e., Nash equilibrium, where each policymaker maximizes its own country's welfare, taking as given the policy of the other policymaker. This non-cooperative scenario is taken as a benchmark to evaluate whether policy cooperation raises or reduces welfare. We then consider two candidate forms of cooperation: the decentralized monetary cooperation and the delegated monetary coordination. In the environment of a decentralized monetary cooperation, there exists a mutually agreed commitment between the two policymakers and the commitment regards money supply. We specify that Home and Foreign agree to set a fixed exchange rate at $\bar{E}$, and the exchange rate depends on Home and Foreign's relative money innovations. In this scenario, each policymaker fully controls its monetary authority and both countries act in the same way: there is no leader and the relative money innovation is simultaneously determined according to their best responses. We show that the cooperative equilibrium allocation is more efficient than the Nash equilibrium allocation, because the former reaches the competitive equilibrium allocation but the latter does not.

However, the cooperation is generally not sustainable because each policymaker has an incentive to violate the agreement and changes its monetary innovation in order to raise its national welfare. Without solving the credibility problem for policy cooperation, either of the two countries can not reap the welfare gains from monetary cooperation. In the delegated monetary coordination each country must forgo its monetary authority so as to overcome the credibility problem. Delegated mechanism is more stringent than the decentralized agreement in the sense that each country's policymaker has no way to change its domestic money supply unless it withdraws from the delegated coordination. We show that, under certain conditions, the delegate is able to set up a monetary rule which attracts both countries to participate in the coordination voluntarily.

empirical analysis, see Feenstra et al. (1996) and Yang (1997).
We show that when the extent of internationalized production increases, the room for cooperative agreement among monetary authorities to improve the welfare of both countries becomes more restricted. Since sustaining a cooperative agreement is generally difficult as demonstrated by Benigno (2002), we analyze the role of a delegated monetary authority that assigns each country an optimal weight to maximize world welfare. The main finding of the paper is that the ranges of parameters that support the viability of a delegate as well as the sizes of welfare gains from policy coordination shrink substantially as the extent of internationalized production increases. Therefore, trading in intermediate goods is able to exploit the welfare gains and eventually replace the need for an international monetary authority. Furthermore, we show that policy coordination for economies with a larger difference in “economic sizes” (the share of domestically-produced final goods in world consumption) becomes less desirable when the extent of internationalized production rises.

The intuition why internationalized production matters for international monetary coordination is as follows. In our model each country produces a tradeable final good. Each worker produces a specific brand of intermediate goods which is either domestically-used or exported to produce final goods. We say that an agent works in a particular final-goods sector if she produces the intermediate good for that sector. The key is that internationalized production introduces an alternative channel in which trading in intermediate goods allows an agent’s welfare to depend on the final-goods sector she is associated with, rather than on her nationality. Thus, changes in terms of trade due to a monetary expansion deteriorate the purchasing power of agents who produce intermediate goods for domestic final goods sector, while they improve the purchasing power of the rest of the population. The latter effect induces a more expansionary monetary shock than when internationalized production is not present, and thus it alleviates the contractionary bias. In addition, even though these two countries are of the same size in population, their fractions of population allocated to each final-goods sector are generally different, which determine the degrees of impact of monetary policy on national welfare. This explains why the extent of internationalized production affects national welfare and thus affects the welfare gains from monetary policy coordination.

Some recent works have tackled the issue of international monetary coordination in the new-open-macroeconomics framework. For example, Obstfeld and Rogoff (2002) find that, unless risk aversion is very high, the
potential gains from international monetary coordination is quite limited. On the other hand, Benigno (2001) shows that the potential gains of policy coordination can be large when international financial markets are imperfect. Finally, Pappa (2004), using a more general specification of preferences, shows that gains from policy cooperation in general depend on the degree of openness of the two economies, elasticity of substitution between home and foreign goods, and intertemporal elasticity of substitution. In this paper our numerical examples demonstrate that, without internationalized production, the size of welfare gains from policy coordination depends on model parameters and can be large when elasticity of substitution between home and foreign goods rises. The welfare gains, however, quickly diminish and eventually become nil when the extent of internationalized production increases. This paper thus contributes to this line of research by suggesting that international trade in intermediate goods, as has been more and more prevalent, undermines the viability of an international monetary authority.

The paper is structured as follows. Section 2 outlines the environment of the model and solves for the equilibrium outcomes. Section 3 analyzes the Nash equilibrium and the credibility of a decentralized cooperative arrangement. Section 4 characterizes the condition under which delegated monetary coordination is sustainable. Numerical examples are provided to demonstrate how the feasibility set for coordination and welfare gains change when internationalized production becomes more prevalent. Section 5 concludes.

2 The Model

There are two countries, Home and Foreign, consisting of a national population defined over a continuum of unit mass and specializing in the production of final goods \( x \) and \( y \), respectively. Final good production utilizes distinct brands of intermediate goods as input and labor is the only input for producing intermediate goods. Each consumer is endowed with either type-\( x \) or type-\( y \) labor service which is specific to manufacturing a distinct brand of intermediate goods for producing final goods \( x \) and \( y \), respectively. Consumption-good markets are perfectly competitive, while intermediate-good markets are monopolistically competitive. All goods are tradable without barriers across countries. The nominal price of an intermediate good is predetermined at the end of the previous period and contracted in terms of the buyer’s currency.
In the following, we give definitions of notations to be used throughout the paper. Given generic variables $N_h$ for home country and $N_f$ for foreign country, we define world variables as $N_W = (N_h)^\gamma (N_f)^{1-\gamma}$ while relative variables are defined as $N_R = N_h / N_f$. The initial equilibrium is indexed by a subscript 0 (such as $N_R^0$), the long run equilibrium is indexed by upperbars (such as $\bar{N}_R$), and the short-run equilibrium contains non-indexed plain variables (such as $N_R$).

Let a fraction $\theta_{jk}^h$ of the labor force be type-$k$ in a country $j$, where $k = x, y$ and $j = h, f$. The aggregate production function of Home at date $t$ is given by

$$x_h = \left( \int_0^{\theta_{hx}^h} I_{hx}^h(z)^{\phi-1} \frac{d\phi}{\phi} \right) \phi/(\phi - 1) + \left( \int_0^{\theta_{fx}^f} I_{fx}^f(z)^{\phi-1} \frac{d\phi}{\phi} \right) \phi/(\phi - 1),$$

where $x$ is the aggregate output of Home; $I_{hx}^h(z)$ and $I_{fx}^f(z)$ are respectively the amount of intermediate good produced by a type-$x$ agent $z$ in Home and Foreign; and $\phi$ denotes the elasticity of input substitution, $\phi > 1$. In the intermediate-goods sector, one unit of labor produces one unit of intermediate good. Thus, $I_{hx}^h(z)$ and $I_{fx}^f(z)$ also represent the quantities of labor supplied. The production technology in Foreign is similarly specified. Given the price of final good $k$, $p_{jk}^l$, and the prices of intermediate goods by agent $z$, $q_{jk}^l(z)$, a firm chooses the amounts of intermediate goods to maximize its profit.

The lifetime utility of an agent $z$ with type-$k$ labor in country $j$ is represented by

$$U_{jk}^l(z) = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_{jk,t}^l(z)^{1-\rho}}{1-\rho} + \chi \ln \frac{m_{jk,t}^l(z)}{p_{jk}^l} - \frac{a}{2} I_{jk,t}^l(z)^2 \right],$$

where $\beta$ is the discount rate, $1/\rho$ is the elasticity of intertemporal substitution, $m_{jk,t}^l(z)$ is the amount of the money holding, and $c_{jk,t}^l(z)$ is the consumption index defined over the consumption of good $x$, $x_{jk,t}^l(z)$, and the consumption of good $y$, $y_{jk,t}^l(z)$, according to $c_{jk,t}^l(z) \equiv x_{jk,t}^l(z)^\gamma y_{jk,t}^l(z)^{1-\gamma}$.

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$^3$Table A in the appendix and the last column in Table 1 summarize the main notations used in the paper.
0 < γ < 1. The corresponding consumption-based price indices \( p^j \) are determined as
\[
0 < \gamma < 1. \quad \text{The corresponding consumption-based price indices } p^j \text{ are determined as } p^j = (p^j_x)^{1-\gamma} / \gamma^\gamma (1-\gamma)^{1-\gamma}. \text{ Markets are not segmented so that the law of one price holds, } E p^f = p^h, \text{ where } E \text{ is the exchange rate.}
\]

Agents hold two assets, national money \((m)\) and an international bond \((b)\). A Home type-\(x\) agent’s budget constraint is as follows and the analog for other agents are similar:
\[
b_{x,t+1}^h(z) + m_{x,t}^h(z) - m_{x,t-1}^h(z) \leq (1 + i_t^h) b_{x,t}^h(z) + p_{x,t}^h(z) I_{x,t}^h(z) - p_{h,c}^h(x,t),
\]
where \(i_t^h\) is the nominal yield of bonds issued by Home. An agent, taking prices of goods and demand for intermediate goods as given, chooses \(x_{x,t}^h, y_{x,t}^h, c_{x,t}^h, m_{x,t}^h, b_{x,t+1}^h,\) and \(I_{x,t}^h\) to maximize her lifetime utility.

The world resource constraints require that the aggregate output is no less than the world consumption for any good \(k\), \(k = x\) if \(j = h\); and \(k = y\) if \(j = f\):
\[
k_t \geq \int_0^{\theta_{x,t}^h} k_{x,t}^h(z) dz + \int_{\theta_{x,t}^h}^{1} k_{x,t}^h(z) dz + \int_0^{\theta_{y,t}^f} k_{x,t}^f(z) dz + \int_{\theta_{y,t}^f}^{1} k_{x,t}^f(z) dz,\]

and the international bond is in zero net supply, \(b_t^h + b_t^f = 0\), where \(b_t^h = \theta_t^h b_{x,t}^h + \theta_t^f b_{y,t}^f\). The solution to this model derived in the Appendix A is summarized in Table 1.

In the equilibrium, no agent will supply labor service to an extent that the marginal benefit from working is less than the marginal cost from working. Accordingly, we can write the participation conditions of consumers as
\[
q_{k,t}^j / p^j \geq a I_{k,t}^j (c_{k,t}^j)^{\rho}, \text{ which by Table 1 can also be expressed as } c_{k,t}^{1-\rho} \geq a \theta_k^{2b/(1-\rho)} k^2, k = x, y, \text{ where } \theta_k \equiv \theta_k^h + \theta_k^f \text{ is the world population working in sector } k. \text{ When all the participation conditions are binding, the world economy reaches a competitive equilibrium allocation.}
\]

### 3 International Monetary Policy

In this section we first derive the monetary policy in a Nash equilibrium as a benchmark and then analyze the prospects and effects of decentralized monetary coordination when internationalized production matters.
Table 1: Solution of the Model

<table>
<thead>
<tr>
<th>long-run equilibrium</th>
<th>short-run equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{p}_h/\tilde{p}_f = \Theta \left( \gamma \left( \frac{1}{1+\delta} - \rho \right) \right) \left( \frac{1+\rho}{1+\rho - \rho} \right) )</td>
<td>( p_h^0/p_f^0 = n_8 C^{-1} \tilde{M}_R^{-1} ) terms of trade</td>
</tr>
<tr>
<td>( \tilde{x} = n_1 (\gamma (1-\gamma^{-1}) A_{\gamma} )</td>
<td>( x = n_8 C^{-1-\gamma} A_{\gamma} \tilde{M}_R^{-1-\gamma} ) output of ( x )</td>
</tr>
<tr>
<td>( \tilde{y} = n_1 (\gamma (1-\gamma^{-1}) A_{\gamma} )</td>
<td>( y = n_7 C^{-1-\gamma} A_{\gamma} \tilde{M}_R^{-1-\gamma} ) output of ( y )</td>
</tr>
<tr>
<td>( \tilde{c}_x = \tilde{c}<em>f = n_3 (\gamma (1-\gamma^{-1}) A</em>{\gamma} )</td>
<td>( c_x = c_f = n_5 \tilde{M}_R^{1-\gamma} ) type-x consumption</td>
</tr>
<tr>
<td>( \tilde{c}_y = \tilde{c}<em>f = n_3 (\gamma (1-\gamma^{-1}) A</em>{\gamma} )</td>
<td>( c_y = c_f = n_5 (\gamma (1-\gamma^{-1}) \tilde{M}_R^{1-\gamma} ) type-y consumption</td>
</tr>
<tr>
<td>( \tilde{m}_k/\tilde{p}_f = \chi \delta^{-1} (1+\delta) )</td>
<td>( m_k/p_f = \chi \delta^{-1} (1+\delta) c_k^{0} ) real balances</td>
</tr>
<tr>
<td>( \tilde{E} = \left( \begin{array}{c} \delta^{h} + \delta^{f} \end{array} \right) )</td>
<td>( \tilde{E} = C M_R ) exchange rate</td>
</tr>
<tr>
<td>( \rho^0_h n_4 \tilde{M}_h \left[ \delta^h + \delta^f \right) \left( \tilde{E}^{-1} \right) \left( \tilde{c}_k^{0} \right) )</td>
<td>( \rho^0_h = n_4 \tilde{M}_h \left[ \delta^h + \delta^f \right) \left( \tilde{E}^{-1} \right) \left( \delta^f \right) ) intermediate good</td>
</tr>
<tr>
<td>( 1+\tilde{r} = \beta^{-1} \delta^{h} c_k^{0} )</td>
<td>( 1+\tilde{r} = \beta^{-1} \delta^{h} c_k^{0} ) real interest rate</td>
</tr>
</tbody>
</table>

Here, \( j = h, f, k = x, y, \Phi = (\phi - 1)(\omega \phi)^{-1}, \gamma = (1-\gamma)^{-1}, \Theta = \Theta_{\gamma} \Theta_{\gamma}, N_R = N^h/N^f \) and \( N_w = \left( N^h \right)^{-1} \) for a variable \( N, A = \Theta_{\gamma} + \Theta_{\gamma} \left( \Theta_{\gamma} \right)^{-1} \), \( C = \left( \Theta_{\gamma} + \Theta_{\gamma} \left( \Theta_{\gamma} \right)^{-1} \right) \left( \Theta_{\gamma} + \Theta_{\gamma} \left( \Theta_{\gamma} \right)^{-1} \right)^{-1} \), and \( n_1 = \Phi \left( \begin{array}{c} \frac{1-\gamma}{\gamma} \end{array} \right) \), \( n_2 = \Theta_{\gamma} \left( \begin{array}{c} \frac{1-\gamma}{\gamma} \end{array} \right) \), \( n_3 = \Theta_{\gamma} \left( \begin{array}{c} \frac{1-\gamma}{\gamma} \end{array} \right) \), \( n_4 = \delta \gamma_{\chi} \chi^{-1} (1+\delta) \), \( n_5 = \tilde{M}_{w0} \), \( n_6 = (\chi \chi_{\gamma}) \), \( n_7 = \left( 1-\gamma \right) \gamma_{\omega} \), \( n_8 = \tilde{p}_x^0/p_{f0}^0 \).
3.1 Monetary Policy in a Nash Equilibrium

Since the long-run equilibrium consumption and production are independent of monetary innovations, the national welfare \( U_j \) is defined as the aggregation of the short-run individual welfare, where

\[
U_j = \sum_{k=x,y} \theta_j^k \left( \frac{c_k}{1 - \rho} - \frac{a}{2} \theta_j^{x+y} k^2 \right), \quad j = h, f.
\]

In Home the monetary stock \( M^h \) chosen to maximize welfare \( U^h \) is determined by the first-order condition,

\[
\theta_x^h \frac{dc_x}{dM^h} \left[ c_x^{1-\rho} - a \theta_x^{x+y} x \left( 1 + \frac{1 - \gamma / \rho}{\gamma} \right) \frac{dx}{dc_x} \right]
+ \theta_y^h \frac{dc_y}{dM^h} \left[ c_y^{1-\rho} - a \theta_y^{x+y} y (1 - \rho) \frac{dy}{dc_y} \right] = 0. \tag{1}
\]

The terms in the first pair of square brackets show the marginal utility of consumption, \( c_x^{-\rho} \), minus the disutility of working in exchange for one unit of the composite consumption, \( a \theta_x^{x+y} x (1 + \frac{1 - \gamma / \rho}{\gamma} \rho) dx/dc_x \), for a type-\( x \) agent. The disutility of working plays a very important role in our analysis. This level of disutility is increasing in \( \rho \) (the substitutability between goods \( x \) and \( y \)), \( \phi \) (the elasticity of input substitution), and \( (1 - 4 \) in the long run money is neutral. This key feature comes from the fact that the elasticity of relative net output demand with respect to relative price is equal to one. Thus, given initially a zero current account in each country, when there is a monetary shock the current account variation induced by a change in relative net output demand and that induced by a change in relative price offset each other, so that the current account remains zero in each country. Since neither country is a net debtor in equilibrium, individuals’ debt positions do not affect their consumption and work decisions in equilibrium. Thus, when all prices respond to shocks in the long run, the economy returns to the initial steady state. See Chu (2005) for details.

Real balance holding provides liquidity services and thus generates utilities for agents. This is a pragmatic way of modelling money to yield positive prices for money. Corsetti and Pesenti (2001) show that omitting real balance holding will not affect the level of welfare unless the coefficient attached to real money balance, \( \chi \), is very large. Thus, we can consider \( \chi \) to be a small number. When we compute the welfare level, we neglect the real balance holding by assuming that the coefficient \( \chi \) is sufficiently small and emphasize the consumption of goods and leisure. Obstfeld and Rogoff (1998), Devereux and Engel (1999a), Benigno (2002) and Chu (2005) apply the same approach.

If \( \rho > 1 \), i.e., the intertemporal elasticity of consumption, \( 1/\rho \), is less than the elasticity of intratemporal substitution between good \( x \) and good \( y \), which is one, then the higher
the relative share of total expenditure on the Foreign good to Home good), and decreasing in $\theta_x$ (the size of sector $x$ in terms of its working population).

To explain these results, recall that when the relative price of $x$ to $y$ decreases after Home’s monetary expansion, the market demand for good $x$ surges and type-$x$ agents of the Home country increase their labor supply to meet the market demand. First, the larger the substitutability is between goods $x$ and $y$ ($\rho$ is larger), the higher the demand is for $x$, which means each type-$x$ agent has to work more and therefore endure more disutility. Second, when the elasticity of input substitution is larger ($\phi$ is larger) the monopolistic distortion in production is smaller - that is, the marginal productivity of labor is lower. Thus, to meet the increased market demand for $x$ each type-$x$ agent also has to work more. Third, if Foreign goods represent a larger share in world consumption ($\left(1 - \gamma\right)/\gamma$ is larger) and the relative price of Foreign goods is also higher (due to changes in terms of trade), then an agent has to work more to acquire the same level of consumption. Finally, if there are fewer workers in sector $x$ ($\theta_x$ is smaller), then each worker needs to work more to keep up with the higher market demand.

Similar arguments go through for the terms in the second pair of square brackets in (1), whereby the disutility of working in exchange for one unit of the composite consumption for a type-$y$ agent, $a\theta^\phi/(1-\phi) \gamma (1-\rho) dy/dc_y$, is increasing in $\phi$, but decreasing in $\theta_y$ and in $\rho$ for $\rho < 1$. Note that if $\rho > 1$ and thus goods $x$ and $y$ are substitutes, then demand for $y$ decreases after a shock. This says that only when $\rho$ is less than one do type-$y$ agents need to work more to meet market demand. The higher the complementarity is between $x$ and $y$ ($\rho$ is smaller), the more the demand for $y$ is raised. Note that the ratio $\left(1 - \gamma\right)/\gamma$ is not included the brackets, because the change in demand for $y$ is aligned to the change in demand for $x$ which will respond to $\left(1 - \gamma\right)/\gamma$.

Plugging the solution in Table 1 into the first-order conditions (1), we get

demand for good $y$ in the intertemporal adjustment is dominated by the lower demand for $y$ in the intratemporal adjustment. Therefore, $\rho > 1$ implies that world demand for good $y$ decreases by a shock that raises the world demand for $x$, and we say good $y$ and good $x$ are substitutes. If $\rho < 1$, then good $y$ and good $x$ are complements.
\[ c_{x}^{1-\rho} \left[ \theta_{x}^{h} + \theta_{y}^{h} (\Theta) (\Psi)^{\rho-1} \right] = \frac{a \theta_{x}^{\gamma}}{c_{x}^{1-\rho}} x^{\gamma} \left[ \theta_{x}^{h} \left( 1 + \frac{1 - \gamma \rho}{\Psi} \right) \right. \]
\[+ \theta_{y}^{h} (1 - \rho) \left( \frac{M_{R0}}{M_{R}} \right)^{2} (\Psi)^{\rho-1} \right], \quad (2) \]

where \( \Psi \equiv \gamma / (1 - \gamma) \), \( \Theta \equiv (\theta_{x}^{h} + \theta_{y}^{f}) / (\theta_{x}^{h} + \theta_{y}^{f}) \), and \( \Psi \Theta = d / d \epsilon = c_{x} / c_{y} \) by Table 1. The left-hand side of equation (2) is the aggregate marginal welfare gains from consumption and the right-hand side is the aggregate marginal welfare losses from working.

Similarly, the monetary stock \( M_{f} \) is chosen in Foreign and, by the first order condition, we can write

\[ c_{x}^{1-\rho} \left[ \theta_{x}^{f} + \theta_{y}^{f} (\Psi) (\Psi)^{\rho-1} \right] = \frac{a \theta_{x}^{\gamma}}{c_{x}^{1-\rho}} x^{\gamma} \left[ \theta_{x}^{f} (1 - \rho) + \theta_{y}^{f} \left( 1 + \frac{\gamma}{1 - \gamma} \rho \right) \right. \]
\[\left. \left( \frac{M_{R0}}{M_{R}} \right)^{2} (\Psi)^{\rho-1} \right]. \quad (3) \]

Using the ratio of (2) to (3), we solve for the relative unanticipated money innovation \( \bar{M}_{N}^{R} \) under Nash equilibrium, where the superscript \( N \) denotes Nash equilibrium,

\[ \frac{\bar{M}_{N}^{R}}{M_{R0}} = \Delta_{N}, \]

where

\[ \Delta_{N} = \frac{\left( \frac{\theta_{x}^{h} + \theta_{y}^{h} (\Psi) (\Psi)^{\rho-1}}{\theta_{y}^{f} + \theta_{y}^{f} (\Psi)^{\rho-1}} \right) - \left( \frac{\theta_{x}^{h}}{\theta_{y}^{f}} \right) \theta_{y}^{h} \theta_{x}^{h} \theta_{y}^{h} (\Psi) (\Psi)^{\rho-1}}{(\Psi)^{1-\rho}} \left[ \left( \frac{1 + \frac{\gamma}{1 - \gamma} \rho}{1 + \frac{\gamma}{1 - \gamma} \rho} \right) \frac{\theta_{y}^{f}}{\theta_{y}^{f}} - \left( \frac{\theta_{x}^{h} + \theta_{y}^{h} (\Psi) (\Psi)^{\rho-1}}{\theta_{y}^{f} + \theta_{y}^{f} (\Psi) (\Psi)^{\rho-1}} \right) \left( \frac{1 + \frac{\gamma}{1 - \gamma} \rho}{1 + \frac{\gamma}{1 - \gamma} \rho} \right) \frac{\theta_{y}^{f}}{\theta_{y}^{f}} \right]}. \]

To find out whether the Nash equilibrium allocation is Pareto efficient, we compute the marginal social welfare \( \partial U / \partial M_{j}, j = h, f \), leaving \( \bar{M}_{N}^{R} = M_{R} \) and evaluating it at the Nash equilibrium. The sign of marginal social welfare is determined by
Internationalized Production and Monetary Policy Coordination

\[ \text{sign} \left( \frac{\partial U^j}{\partial M^j} \right)_{M_R=M^N_R} = \text{sign} \left[ \theta^j_x \left( c^{1-\rho}_x - a\theta^{26}_x \chi^2 \right) + \theta^j_y \left( c^{1-\rho}_y - a\theta^{26}_y \psi^2 \right) \right] = \text{sign} \left[ \theta^j_x (1 - N) + \theta^j_y (\Upsilon \Theta)^{\rho-1} \right. \\
\left. (1 - N \Delta^{-1}_N) \right], \quad (4) \]

where

\[ N = \frac{\Delta_N \left( \theta^h_x + \theta^h_y (\Upsilon \Theta)^{\rho-1} \right)}{\Delta_N \left( 1 + \frac{1-\rho}{\chi \psi} \right) \theta^h_x + (\Upsilon \Theta)^{\rho-1} (1 - \rho) \theta^h_y}. \]

Since the participation conditions are

\[ c^{1-\rho}_x \geq a\theta^{26}_x \chi^2 \quad \text{and} \quad c^{1-\rho}_y \geq a\theta^{26}_y \psi^2, \quad (5) \]

condition (4) must be positive or zero, and it also yields the same sign for both countries. Thus, only when the world economy is at the competitive equilibrium allocation where both constraints in (5) are binding will the Nash equilibrium achieve Pareto efficiency. Equivalently, only when

\[ N = 1 \quad \text{and} \quad \Delta_N = 1 \quad (6) \]

will the Nash equilibrium be Pareto efficient and achieve the competitive allocation. In particular, the condition (6) holds when \( \theta^j_k = \gamma = 0.5 \).

Note that when \( \Delta_N = 1 \), the terms of trade are independent of monetary shocks and the distortion associated with openness itself does not exist. In this case, we return to the closed-economy result in which monopolistic distortions can be eliminated by monetary policy. When \( N = 1 \), the Nash equilibrium is also consistent with welfare maximization and no monetary authority intends to deviate from the equilibrium.

When there is no internationalized production, condition (4) is simplified to \( \text{sign} \left[ \Upsilon^k \rho / (1 + \Upsilon^k \rho) \right] \), where \( k = x, \lambda = -1 \) when \( j = h \), and \( k = y, \lambda = 1 \) when \( j = f \). Since the sign is strictly positive for both countries, the Nash equilibrium allocation can never be a competitive allocation. When there exists internationalized production, an expansionary monetary shock results in lower purchasing power for a part of domestic agents, but higher for the others. The latter effect induces a more expansionary monetary shock than when there is no internationalized production.
and thus alleviates contractionary bias. If condition (6) is satisfied, then the monetary innovations are large enough to eliminate the contractionary bias. An important implication of the our result is that internationalized production provides another channel for approaching Pareto efficiency and competitive allocation, on top of monetary policy cooperation emphasized by the literature.

3.2 Decentralized Monetary Cooperation

Consider that Home and Foreign agree upon an international monetary policy cooperation to set a fixed exchange rate at $\bar{E}$. Country $j$ maximizing its national welfare with respect to its monetary innovation yields the first-order condition

$$\frac{\partial U^j}{\partial M^j_{E=E}} = \theta^j_x \left( c_1^{1-\rho} - a\theta^j_x \frac{\bar{E}}{x^2} \right) + \theta^j_y \left( c_1^{1-\rho} - a\theta^j_y \frac{\bar{E}}{y^2} \right) = 0,$$

$j = h, f.$

It is clear that in equilibrium both participation constraints must be binding. Therefore, this arrangement yields a competitive equilibrium allocation. The optimal relative monetary innovation under the cooperative arrangement $\bar{M}^C_R$, where the superscript $C$ denotes cooperative arrangement, is given by

$$\bar{M}^C_R = M^R_0.$$ (7)

Because the exchange rate and prices are rigid, we again restore the efficient allocation. Given this cooperative solution we now check whether this cooperative arrangement is sustainable.

Consider the situation whereby country $j$ chooses $M^j$ to maximize its national welfare subject to the constraint that the other country $i$ takes the cooperative arrangement as given:

$$\max \sum_{k=x,y} \theta^j_k \left( c_1^{1-\rho} - a\theta^j_k \frac{\bar{E}}{x^2} k^2 \right)$$

s.t. $\theta^i_x \left( c_1^{1-\rho} - a\theta^i_x \frac{\bar{E}}{x^2} \right) + \theta^i_y \left( c_1^{1-\rho} - a\theta^i_y \frac{\bar{E}}{y^2} \right) = 0.$ (8)

We evaluate the first-order conditions at the competitive solution and the signs of the first-order conditions are respectively given by

$$\text{sign} \left[ \theta^h_y (\Upsilon \Theta)^{0-1} - \theta^h_x \Upsilon^{-1} \right] \quad \text{and} \quad \text{sign} \left[ \theta^f_y (\Upsilon \Theta)^{1-\rho} - \theta^f_x \Upsilon \right].$$ (9)
According to (9), only if the structure of the world economy satisfies
\[
\frac{\theta^h_y (\Upsilon \Theta)^{\rho - 1}}{\theta^h_x} = \frac{\theta^f_y (\Upsilon \Theta)^{\rho - 1}}{\theta^f_x} = \frac{1 - \gamma}{\gamma}
\] (10)
will the arrangement be sustainable whereby, in both countries, the ratio of marginal welfare gains from consumption of type-\(y\) agents to that of type-\(x\) agents equals the relative economic size of Foreign to Home. When there is internationalized production, some domestic agents can enjoy more consumption for a given labor supply due to a higher purchasing power after a shock, while the other agents suffer from deteriorating terms of trade. If the above gains and losses from variations in purchasing power within a country can be cancelled out at this equilibrium, then national welfare maximization is consistent with the objective of international cooperation.

If there is no internationalized production, then condition (9) becomes \(\text{sign}(-\Upsilon)\) for both countries and therefore a decentralized cooperative monetary arrangement can never be sustainable. This is because a country has an incentive to deviate from the designated monetary policy by contracting money supply to improve the terms of trade so that its marginal welfare gains from consumption outweigh its marginal welfare losses from working due to an improved purchasing power.

When decentralized cooperation is not consistent with condition (10), the country with a non-zero sign of the first-order condition (9) has an incentive to violate the agreement in order to increase its national welfare. The cooperation eventually then breaks down.

### 4 Delegated Monetary Coordination

To resolve the credibility problem of policy cooperation, we consider a delegate who commits to coordinate and enforce international monetary policy. A country is willing to give up its independence of monetary policy only if the centralized arrangement does not make it worse off than under the non-cooperative allocation. Given that both countries must be no worse off than in the Nash equilibrium, we investigate how internationalized production affects the role of the delegate. To proceed, we first characterize the Pareto frontier which is derived by choosing national monetary innovations to maximize the weighted average welfare function for each given value of welfare weight. The delegate then chooses the optimal welfare weight sub-
ject to the constraints that both countries must be no worse off than under Nash equilibrium.

The Pareto frontier is characterized by

$$W = \alpha U^h + (1 - \alpha) U^f,$$

where $$\alpha \in [0, 1]$$ is the weight attributed to the Home welfare. We show in Appendix C that the allocation on the Pareto frontier is consistent with a competitive equilibrium allocation only when

$$\frac{\theta_{ax}(\gamma \Theta)^{\rho - 1}}{\theta_{ax}} \equiv \alpha \frac{\theta_{bx}(\gamma \Theta)^{\rho - 1}}{\theta_{bx}} + (1 - \alpha) \frac{\theta_{fy}(\gamma \Theta)^{\rho - 1}}{\theta_{fy}} = \frac{1 - \gamma}{\gamma}, \quad (11)$$

where $$\theta_{ax} \equiv \alpha \theta_{hx} + (1 - \alpha) \theta_{fx}$$, $$\theta_{ay} \equiv \alpha \theta_{hy} + (1 - \alpha) \theta_{fy}$$. Condition (11) implies that there is a particular $$\alpha$$, given a combination of $$\rho$$, $$\gamma$$, $$\theta_{hx}$$, $$\theta_{fx}$$, $$\theta_{hy}$$, and $$\theta_{fy}$$, at which the Pareto frontier achieves the competitive equilibrium allocation. We refer to the welfare weight that satisfies (11) as the competitive $$\alpha$$ hereafter. Note that the left-hand side of condition (11) is a linear combination of the relative marginal welfare gains from consumption of type-x agents to type-y agents in Home and in Foreign.

By equation (10) we can tell that when $$\alpha = 0.5$$ the delegated coordination is equivalent to the decentralized cooperation. In a delegated coordination the welfare weight assigned to Home may not be a competitive $$\alpha$$, because there may be no such $$\alpha$$ that satisfies (11) as well as renders both countries no worse off than in the Nash equilibrium. As shown in Appendix C, when only the participation constraint of type-x(y) agents is binding in equilibrium, the allocation on the Pareto frontier implies that ($$\theta_{ax}(\gamma \Theta)^{\rho - 1}/\theta_{ax}$$) > (<) (1 - $$\gamma / \gamma$$), and the optimal monetary rule is

$$\frac{M_R}{M_{R0}} = \left[ \frac{1 + \frac{1 + \rho}{2} \gamma}{1 + (\gamma \Theta)^{1 - \rho} \frac{\theta_{ay}(\gamma \Theta)^{\rho - 1}}{\theta_{ax}}} \right]^{1/2}, \quad \frac{\theta_{ax}(\gamma \Theta)^{\rho - 1}}{\theta_{ax}} < \frac{1 - \gamma}{\gamma},$$

$$= \left[ \frac{1 + \frac{1 + \rho}{2} \gamma}{1 + (\gamma \Theta)^{1 - \rho} \frac{\theta_{ay}(\gamma \Theta)^{\rho - 1}}{\theta_{ax}}} \right]^{1/2}, \quad \frac{\theta_{ax}(\gamma \Theta)^{\rho - 1}}{\theta_{ax}} > \frac{1 - \gamma}{\gamma},$$

$$= 1, \quad \text{when } \frac{\theta_{ax}(\gamma \Theta)^{\rho - 1}}{\theta_{ax}} = \frac{1 - \gamma}{\gamma}. \quad (12)$$

7This implies that an applicable transfer program might be necessary to apply a competitive $$\alpha$$ in the delegated coordination, which deserves further investigation in the future.
This says that when the economic size of Home is smaller (γ is smaller) and thus \((\theta_{\alpha_k}(\gamma \Theta)^{\rho-1}/\theta_{\alpha_k}) < (1 - \gamma/\gamma)\), the optimal monetary policy is designated so that the monetary innovation of Home is less expansionary relative to that of Foreign, and vice versa.

Denote \(U_{ak}^j\) to be the national welfare under the delegated coordination for country \(j\) when only the participation constraint of type-\(k\) agents is binding in equilibrium. We then derive the boundaries of \(\alpha\) for both countries by equating country \(j\)'s welfare under the Nash equilibrium, \(U_{N}^j\), to \(U_{ak}^j\) for \(j = h, f\). A centralized coordination is desirable to both countries only when the designated welfare assigned to each country is within or on the boundaries. The delegated social planner then chooses the optimal \(\alpha\), for \(k = x, y\), to maximize

\[
W_{ak} = \alpha U_{ah}^k (1 - \alpha)U_{af}^k,
\]

subject to

\[
U_{N}^j \leq U_{ak}^j, \quad j = h, f.
\]

To illustrate the optimal \(\alpha\), we plot the solution to this problem in Figures 1. Without loss of generality, in the numerical examples we assume that

\[
\frac{\theta_{h}^X}{\theta_{l}^X} = \frac{\theta_{f}^X}{\theta_{l}^X} \equiv \theta,
\]

which means the ratio of workers producing exported intermediate goods to those producing domestically-used intermediate goods is the same for both Home and Foreign. This symmetry simplifies the numerical computations and we need only to concentrate on \(\theta\) as our measure of the extent of internationalized production.

To begin with, Figures 1a to 1f show the optimal \(\alpha\) for each value of \(\gamma\), given various extents of internationalized production \(\theta\) and substitutability between goods \(\rho\). In each sub-figure the bold-solid line represents the locus of optimal \(\alpha\) chosen by the delegate; the solid line represents the locus of competitive \(\alpha\); the dot-dashed (dashed) lines are the locus of \(\alpha\) when the national welfare under the Nash equilibrium equals its national welfare under the delegated coordination for Foreign (Home). The dot-dashed line and dashed line are therefore the boundaries for the optimal \(\alpha\). Both countries are willing to comply with the delegated coordination only if the optimal weight \(\alpha\) is confined within or on the boundaries.
The locus of competitive $\alpha$ divides the plane into two areas: on its right-hand (left-hand) side, the underlying parameters satisfy the condition $(\theta_{\alpha y}(Y)\rho^{-1}/\theta_{\alpha x}) > (<) (1 - \gamma /\gamma)$ and the corresponding lines in this area describe the equilibria when only the participation constraint of type-$x(y)$ agents is binding. The locus of competitive $\alpha$ satisfies the condition (11) under which both participation constraints are binding along the locus. We analyze Figure 1 for different extents of internationalized production $\theta$ and substitutability between goods $\rho$, respectively. We then compute welfare gains from policy coordination given various extents of internationalized production. We consider $\rho = 0.5$ ($x$ and $y$ are complements) and $\rho = 2$ ($x$ and $y$ are substitutes), respectively. Tables 2 and 3 summarize the welfare gains of both countries when monetary policies are coordinated given various parameter values of $\rho$, $\gamma$, and $\theta$. Some interesting findings are worth noting.

First, as shown in Figure 1, when $\theta$ becomes larger, a narrower range of $\gamma$ is available for supporting the optimal monetary coordination, for any given $\rho$. Take $\rho = 0.5$ for example, comparing Figures 1a through 1c, it is clear that when the extent of internationalized production increases the locus of optimal $\alpha$ (the bold-solid line) becomes steeper, and the area between the boundaries, i.e., the feasibility set for coordination, shrinks accordingly. This illustrates the main theme of our paper: internationalized production reduces the room for mutually beneficial policy coordination. The intuition has been discussed in the introduction.

Second, an alternative way to see this is to compute the welfare gains from policy coordination. From Table 2 and 3, it is interesting to observe that when Home and Foreign are of equal economic size ($\gamma = 0.5$), the maximum possible gains from coordination occurs when $\theta = 0$, with merely 3% if $\rho = 0.5$ and can be as large as 11% if $\rho = 2$ for each country. Some recent studies have argued that welfare gains from international policy coordination are, in general, quantitatively small. Here, we show that the size of welfare gains can be large for a higher degree of substitutability between good $x$ and good $y$. The welfare gains from coordination, however, start to decline as $\theta$ rises and completely vanish as the extent of internationalized production reaches its maximum ($\theta = 1$) for any value of $\gamma$. Our numeri-

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8 Even with different approaches, earlier contributions to the question of whether central banks should coordinate their policy also find similar results. See Canzoneri and Henderson (1991) for a representative example of this literature on policy coordination, and also Canzoneri et al. (2003) for an excellent overview.
Figure 1: The Optimal Welfare Weights Given Various Extents of Model Parameters

Note: The solid line is the locus of alpha that implements the competitive allocation (competitive $\alpha$), which divides the plane into two areas. On the right-hand-side, the dot-dashed line and dashed line are the national welfare for country Foreign and Home, respectively, when the participation constraints of type-x agents are binding in equilibrium, while on the left-hand-side, they are the national welfare for country Foreign and Home, respectively, when the participation constraints of type-y agents are binding. Finally, the bold-solid line is the optimal choice of alpha (optimal $\alpha$) by the delegated social planner. The dot-dashed line and dashed line are therefore the boundaries for the optimal $\alpha$. Both countries are willing to comply with the delegated coordination only if the optimal weight $\alpha$ is confined within or on the boundaries.
Table 2: Welfare Gains from Monetary Policy Coordination When Two Goods are Complements ($\rho = 0.5$)

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\theta$</th>
<th>Home Welfare</th>
<th>Foreign Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>Non-cooperative</td>
<td>0.5214</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>Cooperation</td>
<td>133.635</td>
</tr>
<tr>
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<td>0.5</td>
<td>Welfare Gains</td>
<td>3.024%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>132.986</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>133.635</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td></td>
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<td>1</td>
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<td>133.635</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Welfare Gains</td>
<td>3.024%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>132.986</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>133.635</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>0.49</td>
<td></td>
<td>0.5214</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Non-cooperative</td>
<td>127.888</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cooperation</td>
<td>132.349</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Welfare Gains</td>
<td>3.488%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>132.728</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Welfare Gains</td>
<td>2.540%</td>
</tr>
<tr>
<td></td>
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<td>132.728</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>0.475</td>
<td></td>
<td>0.5600</td>
</tr>
<tr>
<td></td>
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<td>Non-cooperative</td>
<td>73.306</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cooperation</td>
<td>76.377</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Welfare Gains</td>
<td>4.190%</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Welfare Gains</td>
<td>1.709%</td>
</tr>
</tbody>
</table>
Table 3: Table 3 Welfare Gains from Monetary Policy Coordination When Two Goods are Substitutes ($\rho=2$)

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\theta = 0$</th>
<th>$\theta = 0.2$</th>
<th>$\theta = 0.5$</th>
<th>$\theta = 0.8$</th>
<th>$\theta = 1$</th>
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</thead>
<tbody>
<tr>
<td>$\gamma = 0.5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Welfare weight ($\alpha$)</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>—</td>
</tr>
<tr>
<td>Home Welfare Non-cooperative</td>
<td>$-1,068.4$</td>
<td>$-1,022.65$</td>
<td>$-978.677$</td>
<td>$-956.612$</td>
<td>$-952.441$</td>
</tr>
<tr>
<td>Cooperative</td>
<td>$-952.441$</td>
<td>$-952.441$</td>
<td>$-952.441$</td>
<td>$-952.441$</td>
<td>$-952.441$</td>
</tr>
<tr>
<td>Welfare Gains</td>
<td>10.854%</td>
<td>6.865%</td>
<td>2.681%</td>
<td>0.436%</td>
<td>0%</td>
</tr>
<tr>
<td>Foreign Welfare Non-cooperative</td>
<td>$-1,068.40$</td>
<td>$-1,022.65$</td>
<td>$-978.677$</td>
<td>$-956.612$</td>
<td>$-952.441$</td>
</tr>
<tr>
<td>Cooperative</td>
<td>$-952.441$</td>
<td>$-952.441$</td>
<td>$-952.441$</td>
<td>$-952.441$</td>
<td>$-952.441$</td>
</tr>
<tr>
<td>Welfare Gains</td>
<td>10.854%</td>
<td>6.865%</td>
<td>2.681%</td>
<td>0.436%</td>
<td>0%</td>
</tr>
<tr>
<td>$\gamma = 0.49$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Welfare weight ($\alpha$)</td>
<td>0.5545</td>
<td>0.5817</td>
<td>0.6634</td>
<td>0.9903</td>
<td>—</td>
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<tr>
<td>Home Welfare Non-cooperative</td>
<td>$-1,028.51$</td>
<td>$-977.843$</td>
<td>$-930.508$</td>
<td>$-906.989$</td>
<td>$-902.508$</td>
</tr>
<tr>
<td>Cooperative</td>
<td>$-896.917$</td>
<td>$-899.009$</td>
<td>$-901.102$</td>
<td>$-902.497$</td>
<td>$-902.508$</td>
</tr>
<tr>
<td>Welfare Gains</td>
<td>12.795%</td>
<td>8.062%</td>
<td>3.160%</td>
<td>0.495%</td>
<td>0%</td>
</tr>
<tr>
<td>Foreign Welfare Non-cooperative</td>
<td>$-995.74$</td>
<td>$-959.61$</td>
<td>$-923.58$</td>
<td>$-905.29$</td>
<td>$-903.96$</td>
</tr>
<tr>
<td>Cooperative</td>
<td>$-909.473$</td>
<td>$-907.381$</td>
<td>$-905.288$</td>
<td>$-903.893$</td>
<td>$-903.962$</td>
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<tr>
<td>Welfare Gains</td>
<td>8.663%</td>
<td>5.443%</td>
<td>1.981%</td>
<td>0.154%</td>
<td>0%</td>
</tr>
<tr>
<td>$\gamma = 0.475$</td>
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<tr>
<td>Welfare weight ($\alpha$)</td>
<td>0.6590</td>
<td>0.7385</td>
<td>0.9769</td>
<td>1.9307</td>
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<tr>
<td>Home Welfare Non-cooperative</td>
<td>$-968.122$</td>
<td>$-911.22$</td>
<td>$-859.782$</td>
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<tr>
<td>Cooperative</td>
<td>$-823.459$</td>
<td>$-827.295$</td>
<td>$-831.131$</td>
<td>$-833.688$</td>
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<tr>
<td>Welfare Gains</td>
<td>14.943%</td>
<td>9.210%</td>
<td>3.332%</td>
<td>0.102%</td>
<td>—</td>
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<tr>
<td>Foreign Welfare Non-cooperative</td>
<td>$-892.766$</td>
<td>$-869.311$</td>
<td>$-843.866$</td>
<td>$-830.629$</td>
<td>—</td>
</tr>
<tr>
<td>Cooperative</td>
<td>$-846.473$</td>
<td>$-842.638$</td>
<td>$-838.802$</td>
<td>$-836.245$</td>
<td>—</td>
</tr>
<tr>
<td>Welfare Gains</td>
<td>5.185%</td>
<td>3.068%</td>
<td>0.600%</td>
<td>$-0.676%$</td>
<td>—</td>
</tr>
</tbody>
</table>

Note: The case with $\gamma = 0.51$ and 0.525 is symmetric to that with $\gamma = 0.49$ and 0.475. The welfare levels are each multiplied by 100.
cal examples thus demonstrate that trading in intermediate goods is able to exploit the welfare gains and eventually replace the need for international policy coordination.

Third, Figure 1 shows that the optimal weight and the competitive weight coincides at 0.5 when Home and Foreign are of the same economic size ($\gamma = 0.5$), and that the delegate assigns a larger weight to Foreign when Foreign is larger ($\gamma < 0.5$), and vice versa. Combined with (12), this says that when the economic size of Foreign is larger, the optimal monetary policy is designated by way of assigning a smaller welfare weight ($\alpha < 0.5$) to Foreign so that the monetary innovation of Foreign is more expansionary relative to that of Home. The intuition is as follows. When $\gamma < 0.5$, individual consumers value the consumption of $y$ more than $x$. The optimal monetary policy is thus designated to induce as much production of $y$ as possible. By assigning a lower welfare weight to Foreign, the delegate mandates the monetary innovation in Foreign to be more expansionary, which reduces the relative price of good $y$ to good $x$ and raises the world relative demand for $y$. This policy induces type-$y$ workers to supply labor services to meet the market demand up to the point where their participation constraints are binding. Since only type-$y$ agents’ participation constraints are binding, Foreign suffers from disutility of working more than Home does. This demonstrates how the delegation of monetary policy actually works: by assigning a smaller welfare weight to Foreign, the monetary innovation in Foreign is more expansionary, and thus its welfare gains from coordination are lower.\(^9\)

Fourth, it is also interesting to observe that when Home and Foreign are more unequal in economic sizes, gains from policy coordination diminish even faster as $\theta$ increases. To illustrate this point, consider the case when $\gamma = 0.475$ in Table 2. It is clear that when $\theta$ rises, the welfare gains of Foreign declines rapidly. Foreign’s welfare gains turn negative ($-0.15\%$) when $\theta = 0.8$, which makes policy coordination impossible. Why does size difference matter? We again consider the case when $\gamma < 0.5$ (Foreign is larger in size than Home and there are more type $y$ workers in Foreign). As explained earlier, in this case the delegate assigns a smaller weight to Foreign, and thus Foreign is mandated to be more expansionary. This leads the rel-

\(^9\)Take Table 2 for example. For any given $\theta$, say, $\theta = 0.2$, when $\gamma$ declines from 0.5 to 0.475, the optimal weight increases from 0.5 to 0.5899 and the welfare gains of Home rise from 1.6\% to 2.3\%, while those of Foreign decline from 1.6\% to 0.7\%. A similar argument goes through when $\gamma$ is larger than 0.5.
ative price of good \( y \) to decline and the demand for good \( y \) to rise. Type \( y \) workers have to work a lot more. Thus, Foreign, where more type \( y \) workers reside, may suffer more disutility from working than the benefit from higher consumption. This is particularly true when the extent of internationalized production is higher because with higher \( \theta \) the contractionary bias has been drastically corrected and these countries are producing at a high level of production. In this case, it is more likely that the Foreign's marginal disutility from working outweighs the marginal benefit from consumption, and thus Foreign will not gain from policy coordination.

Finally, by comparing Tables 2 and 3, it is clear that welfare gains from policy coordination are smaller when \( x \) and \( y \) are complements (\( \rho = 0.5 \)) than when \( x \) and \( y \) are substitutes (\( \rho = 2 \)) for all values of \( \gamma \) and \( \theta \). This says that the viability of a delegate declines in the degree of substitutability between the final goods. To see why, suppose that \( \rho = 2 \) (\( x \) and \( y \) are substitutes). A Home expansionary shock will lowers the relative price of good \( x \), induce a larger demand for good \( x \) over good \( y \), and thus lead to a larger increase in labor supply in \( x \) sector. Since the marginal disutility from working is increasing in labor supply and marginal utility from consumption is decreasing in real income, a higher substitutability thus yields a smaller welfare gains from cooperation.

5 Concluding Remarks

Trading in production factors has risen rapidly in the last two decades. In this paper we explore the following questions: When internationalized production becomes more prevalent, what is the scope for international monetary coordination to achieve welfare gains for both countries? Is there a role for a delegated monetary authority?

We show that the scope for monetary policy cooperation is determined by the degree of substitutability between Home and Foreign goods, the difference in economic sizes, and also the extent of internationalized production. When the extent of internationalized production is larger, the room for mutually beneficial policy cooperation shrinks. Since the decentralized cooperative agreement is not sustainable, we analyze the role of a delegated monetary authority. We show that when internationalized production is more prevalent, the range of the weight available to the delegate for choice as well as the ranges of model parameters feasible to achieve welfare gains become more restricted. Furthermore, even though the sizes of welfare gains
can be large, internationalized production can completely exploit the welfare gains from policy coordination. Finally, policy coordination for economies with a larger difference in “economic size” is less desirable when the extent of internationalized production rises. In other words, when internationalized production becomes more prevalent, policy coordination can sustain only if the economic sizes of these two countries are very similar.

One might conjecture that different industrial structures (for example, the share of manufacture sector vs. service sector) may generate differential impacts in response changes in terms of trade due to monetary shocks. However, recent evidence shows that service sector, which is largely non-tradeable in the past, has undergone a significant growth in outsourcing “tasks.” For example, OECD STAN database shows that during 1993–2003 the real imports of “Other Business Services” (including accounting, business management, and consulting), increased by 41% in Canada, 32% in France, 46% in Germany, 102% in US, and 116% in UK (Grossman and Rossi-Hansberg, 2006). See also Baldwin and Robert-Nicoud (2007), and Wong et al. (2006) for more references. Since tradeable services correspond to the intermediate input in our model, changes in terms of trade caused by monetary shock not only affect manufacturing sector which utilizing imported intermediate goods, but also have a significant effect on service trading.

As mentioned above, in contrast to Benigno (2002) where a centralized coordinator in monetary policy is the only way to achieve competitive allocation, we see that the extent of internationalized production, when adjusted appropriately, could provide another channel for achieving Pareto efficiency. An immediate extension of our paper is to endogenize the extent of internationalized production. In a more elaborate model where the extent of internationalized production is endogenous, the trade policy can be engineered to follow condition (10) under a cooperative arrangement so that the global economy obtains the competitive allocation. This implies that if both countries negotiate a trade policy on intermediate goods trade, then internationalized production can serve as a substitute for international monetary policy coordination to achieve competitive allocation.
Appendix

Table A: Summary of Notations

<table>
<thead>
<tr>
<th>notation</th>
<th>description of the notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta^j_k$</td>
<td>the fraction of type-$k$ labor force in country $j$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>the elasticity of input substitution in final-goods sectors</td>
</tr>
<tr>
<td>$1/\rho$</td>
<td>the elasticity of intertemporal substitution of consumption</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>the share of consumption on good $x$</td>
</tr>
<tr>
<td>$\chi$</td>
<td>the coefficient for real money balance</td>
</tr>
<tr>
<td>$\theta_k$</td>
<td>the size of sector $k$ in terms of its working population.</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>$\equiv \left( \theta^h_y + \theta^f_x \right) / \left( \theta^h_x + \theta^f_y \right)$</td>
</tr>
<tr>
<td>$\Upsilon$</td>
<td>$\equiv \gamma / (1 - \gamma)$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$\equiv \theta^h_y / \theta^h_x = \theta^f_x / \theta^f_y$</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>$\equiv (\phi - 1)(a\phi)^{-1}$, $a &gt; 0$.</td>
</tr>
</tbody>
</table>

Appendix A: Solving the Model

The profit maximization of firms implies

$$I^j_{x,t} = \left( \frac{q^j_{x,t}}{p^x_t} \right)^{-\phi} x, \quad \text{and} \quad I^j_{y,t} = \left( \frac{q^j_{y,t}}{p^y_t} \right)^{-\phi} y.$$  

Let $\lambda_k^j$ denote the Lagrange multiplier for type-$k$ agents in country $j$. The first-order conditions of utility maximization for type-$k$ agents in country $j$ are

$$\gamma \left( c^j_{k,t} \right)^{1-\rho} \left( x^j_{k,t} \right)^{-1} = \lambda^j_{k,t} p^j_{k,t},$$

$$\left( 1 - \gamma \right) \left( c^j_{k,t} \right)^{1-\rho} \left( y^j_{k,t} \right)^{-1} = \lambda^j_{k,t} p^j_{y,t},$$

$$\left( c^j_{k,t} \right)^{-\rho} = \lambda^j_{k,t} p^j_t,$$

$$\chi \left( m^j_{k,t} \right)^{-1} = \lambda^j_{k,t} - \beta \lambda^j_{k,t+1},$$

$$\lambda^j_{k,t} = \beta \left( 1 + i^j_{t+1} \right) \lambda^j_{k,t+1}.$$
\[
\left( I^h_{x,t} \right)^{\frac{1}{\phi}} = (\phi - 1)(\alpha \phi)^{-1} k^h_{x,t} p^h_{x,t} x_t, \quad \text{and} \quad \left( I^f_{y,t} \right)^{\frac{1}{\phi}} = (\phi - 1)(\alpha \phi)^{-1} k^f_{y,t} p^f_{y,t} y_t.
\]

Zero profit conditions imply that
\[
p^h_{x,t} x_t = p^h_{x,t} \left( \int_0^{\theta^h_{x,t}} I^h_{x,t}(z) \frac{\phi - 1}{\phi} \, dz + \int_{\theta^h_{x,t}}^{\theta^f_{x,t}} I^f_{x,t}(z) \frac{\phi - 1}{\phi} \, dz \right)^{\frac{\phi}{\phi - 1}} = \theta^h_{x,t} q^h_{x,t} p^h_{x,t} x_t + \theta^f_{x,t} q^f_{x,t} I^f_{x,t},
\]

and
\[
p^f_{y,t} y_t = p^f_{y,t} \left( \int_0^{\theta^f_{y,t}} I^f_{y,t}(z) \frac{\phi - 1}{\phi} \, dz + \int_{\theta^f_{y,t}}^{\theta^f_{y,t}} I^f_{y,t}(z) \frac{\phi - 1}{\phi} \, dz \right)^{\frac{\phi}{\phi - 1}} = \theta^h_{y,t} q^h_{y,t} p^h_{y,t} y_t + \theta^f_{y,t} q^f_{y,t} I^f_{y,t}.
\]

Individual budgets imply that
\[
\theta^h_{x,t} q^h_{x,t} I^h_{x,t} - \theta^h_{x,t} \left( m^h_{x,t} - m^h_{x,t-1} \right) - \theta^h_{x,t} p^h_{x,t} c^h_{x,t} = \theta^h_{x,t} \left( b^h_{x,t+1} - (1 + i_t) b^h_{x,t} \right),
\]
\[
\theta^f_{x,t} w^f_{x,t} I^f_{x,t} / E_t - \theta^f_{x,t} \left( m^f_{x,t} - m^f_{x,t-1} \right) - \theta^f_{x,t} p^f_{x,t} c^f_{x,t} = \theta^f_{x,t} \left( b^f_{x,t+1} - (1 + i_t) b^f_{x,t} \right) / E_t,
\]
\[
\theta^h_{y,t} q^h_{y,t} I^h_{y,t} - \theta^h_{y,t} \left( m^h_{y,t} - m^h_{y,t-1} \right) - \theta^h_{y,t} p^h_{y,t} c^h_{y,t} = \theta^h_{y,t} \left( b^h_{y,t+1} - (1 + i_t) b^h_{y,t} \right),
\]
\[
\theta^f_{y,t} q^f_{y,t} I^f_{y,t} - \theta^f_{y,t} \left( m^f_{y,t} - m^f_{y,t-1} \right) - \theta^f_{y,t} p^f_{y,t} c^f_{y,t} = \theta^f_{y,t} \left( b^f_{y,t+1} - (1 + i_t) b^f_{y,t} \right) / E_t.
\]

Clearing of good markets implies
\[
x_t = \frac{\gamma p^h_{x,t}}{p^h_{x,t}} \left( \theta^h_{x,t} c^h_{x,t} + \theta^f_{x,t} c^f_{x,t} + \theta^h_{y,t} c^h_{y,t} + \theta^f_{y,t} c^f_{y,t} \right),
\]
\[
y_t = \frac{(1 - \gamma) p^f_{y,t}}{p^f_{y,t}} \left( \theta^h_{x,t} c^h_{x,t} + \theta^f_{x,t} c^f_{x,t} + \theta^h_{y,t} c^h_{y,t} + \theta^f_{y,t} c^f_{y,t} \right).
\]
Clearing of debt markets requires
\[ b_t^h + b_t^f = 0, \]
where \( b_t^j = \theta^j b_{x,t}^j + \theta^j b_{y,t}^j \), and clearing of money markets requires
\[ \bar{M}_t^j = \theta^j \bar{m}_x^j + \theta^j \bar{m}_y^j. \]

All of the above equations determine the equilibrium in the world economy where initially \( b_{k,0}^j = 0 \). In the following, the initial equilibrium is indexed by a subscript 0; the short-run equilibrium, where the nominal wages are not able to respond to current shocks, is not indexed; the long-run equilibrium, where the economy reaches a new steady state after shocks, is indexed by upperbars. To solve the long-run equilibrium, we use the above equilibrium conditions and the following six steps: (1) Prove \( i = i^j \), \( I_k = I_k^j, c_k = c_k^j, x_k = x_k^j, y_k = y_k^j, \bar{I}_k = \bar{I}_k^j, \bar{c}_k = \bar{c}_k^j, \bar{x}_k = \bar{x}_k^j, \) and \( \bar{y}_k = \bar{y}_k^j \); (2) Prove \( i = \delta \); (3) Prove \( E = \bar{E} \); (4) Prove \( b^j = \bar{b}^j = 0 \); (5) Solve for \( c_x^j/c_y \) and prove \( c_x^j/c_y = \Upsilon \Theta \); (6) Solve for \( b_k^j \) and \( \bar{b}_k^j \).

Together with the rigidity of the short-run prices and the long-run equilibrium, we derive the short-run equilibrium backwards. The long-run equilibrium and short-run equilibrium are summarized in Table 1. Readers interested in the details of Appendix A are referred to the appendix in Chu (2005).

Appendix B: Solving the Monetary Policy in a Nash Equilibrium and in a Decentralized Coordination

By
\[ U^j = \sum_{k=x,y} \theta^j_k c_k^j \left[ c_k^{1-\rho}/(1-\rho) - a \theta^j_k 2^{1-\phi}(1-\phi) k^2/2 \right], \quad j = h, f, \]
we derive the first-order conditions to be
\[ \frac{\gamma}{\rho M^j} \left\{ \theta^h [c_x^{1-\rho} - ax^2 \theta^h 2^{1-\phi}/(1-\phi) (1 + \frac{1}{\gamma} - \rho)] + \theta^j [c_x^{1-\rho} - ay^2 \theta^j 2^{1-\phi}/(1-\phi) (1 - \rho)] \right\} = 0 \]
and

\[
\frac{1 - \gamma}{\rho M_f} \left\{ \theta_x \left[ c_1^{1-\rho} - ax^2 \theta_x^{2\varphi/(1-\phi)} (1 - \rho) \right] \\
+ \theta_y \left[ c_1^{1-\rho} - ay^2 \theta_y^{2\varphi/(1-\phi)} \left( 1 + \frac{\gamma}{1 - \gamma} \rho \right) \right] \right\} = 0.
\]

From the above two equations and Table 1, we get

\[
\frac{M_R^N}{M_R^0} = \Delta_N^{1/2},
\]

where

\[
\Delta_N = (\gamma \Theta)^{\rho - 1} \left\{ \left[ (\theta_x^b + \theta_y^b (\gamma \Theta)^{\rho - 1}) (1 + (\gamma / 1 - \gamma) \rho) \theta_y^f \\
- (\theta_x^f + \theta_y^f (\gamma \Theta)^{\rho - 1}) (1 - \rho) \theta_y^b \right] / \left[ (\theta_x^f + \theta_y^f (\gamma \Theta)^{\rho - 1}) (1 + (1 - \gamma / \gamma) \rho) \theta_x^b - (\theta_x^b + \theta_y^b (\gamma \Theta)^{\rho - 1}) (1 - \rho) \theta_x^f \right] \right\}.
\]

Plugging \( M_R^N \) into the first-order condition and applying

\[
\begin{equation}
\left( \frac{\gamma}{\chi} \right)^2 = \left( \frac{M_R^0}{M_R} \right)^2 (\Theta^{\varphi - 1} \frac{1}{1 - \gamma}) (\gamma)^{-1},
\end{equation}
\]

we get

\[
\begin{equation}
\frac{c_1^{1-\rho}}{ax^2} = \left( \frac{M_w^0}{M_w} \right)^{\frac{1-\rho}{\varphi}} \left( \frac{M_R^0}{M_R} \right)^{2(1-\gamma)} \left[ \theta_x + \theta_y (\gamma \Theta)^{-1} \right]^{-1} \frac{\phi \theta_x^{(1+\phi)/(1-\phi)}}{(\phi - 1)\gamma},
\end{equation}
\]

and

\[
\begin{equation}
\left( \frac{M_w^N}{M_w^0} \right)^{\frac{1-\rho}{\varphi}} = \frac{\Delta_N^{\gamma} \left( \frac{\theta_x^b + \theta_y^b (\gamma \Theta)^{\rho - 1}}{\theta_x^f + \theta_y^f (\gamma \Theta)^{\rho - 1}} \right) \frac{\phi \theta_x^{(1+\phi)/(1-\phi)}}{(\phi - 1)\gamma}}{\Delta_N \left( 1 + \frac{1 - \gamma / \gamma}{\gamma} \rho \right) \theta_x^b + (\gamma \Theta)^{\rho - 1} (1 - \rho) \theta_y^b}.
\end{equation}
\]

To deal with a decentralized coordination, plugging (13) and (14) into

\[
\sum_{k=x,y} \theta_k \left( c_k^{1-\rho} - a \theta_k^{2\varphi/(1-\phi)} k^2 \right) = 0,
\]

we get

\[
\begin{equation}
\left( \frac{M_w^0}{M_w} \right)^{\frac{1-\rho}{\varphi}} \left( \frac{M_R^0}{M_R} \right)^{2(1-\gamma)} \left[ \theta_x + \theta_y (\gamma \Theta)^{-1} \right]^{-1} \frac{\phi \theta_x^{(1+\phi)/(1-\phi)}}{(\phi - 1)\gamma}
= \theta_x^{2\varphi/(1-\phi)} \left[ \theta_x + \theta_y (M_R^0/M_R)^2 (\gamma \Theta)^{\rho - 1} \right] \left[ \theta_x + \theta_y (\gamma \Theta)^{\rho - 1} \right]^{-1}.
\end{equation}
\]
Thus, the ratio of the two first-order conditions implies that
\[ \bar{M}_C^R = M_{R0}. \]

Plugging \( \bar{M}_C^R \) back into the first-order condition, we get
\[ \left( \frac{M_w^C}{M_w^0} \right)^{1+\rho} = \left( \frac{1}{\theta_x + \theta_y (\Upsilon \Theta)^{-1}} \right) \frac{\phi \theta_x}{(\phi - 1)\gamma}. \]

Appendix C: Solving the Pareto Frontier and Delegate’s Problem

Given \( \bar{M}_R^N \) and \( (M_w^C/M_w^0)^{(1+\rho/\rho)} \), we derive respectively the national welfare of Home and Foreign, \( U_h^N \) and \( U_f^N \), in a Nash equilibrium, which will be the minimum accepted national welfare for participating in a coordination.

We get
\[ U_h^N = c_{x0}^{1-\rho} \left[ \Delta_w \left( \frac{1}{\theta_x + \theta_y (\Upsilon \Theta)^{-1}} \right) \frac{\phi \theta_x}{(\phi - 1)\gamma} \right]^{1+\rho} \]
\[ \times \left[ \theta_x^h \left( \frac{1}{1-\rho} - \frac{1}{2} \Delta_w \Delta_N^{1-\gamma} \right) + \theta_y^h (\Upsilon \Theta)^{\rho-1} \left( \frac{1}{1-\rho} - \frac{1}{2} \Delta_w \Delta_N^{1-\gamma} \right) \right], \]
and
\[ U_f^N = c_{x0}^{1-\rho} \left[ \Delta_w \left( \frac{1}{\theta_x + \theta_y (\Upsilon \Theta)^{-1}} \right) \frac{\phi \theta_x}{(\phi - 1)\gamma} \right]^{1+\rho} \]
\[ \times \left[ \theta_x^f \left( \frac{1}{1-\rho} - \frac{1}{2} \Delta_w \Delta_N^{1-\gamma} \right) + \theta_y^f (\Upsilon \Theta)^{\rho-1} \left( \frac{1}{1-\rho} - \frac{1}{2} \Delta_w \Delta_N^{1-\gamma} \right) \right], \]
where
\[ \Delta_w = \frac{\Delta_N^x (\theta_x^h + \theta_y^h (\Upsilon \Theta)^{\rho-1})}{\Delta_N \left( 1 + \frac{1-\gamma}{\gamma-\rho} \right) \theta_x^h + (\Upsilon \Theta)^{\rho-1}(1-\rho)\theta_y^h}. \]

Given \( \bar{M}_R^C \) and \( (M_w^C/M_w^0)^{(1+\rho/\rho)} \), we derive respectively the national welfare of Home and Foreign, \( U_h^C \) and \( U_f^C \), in a competitive equilibrium,
and we get

$$U^h_C = c_1 x_0 \left( \frac{\phi_2}{\theta_2 (\bar{v}_2) - 1} \right)^{1 - \rho} \left( \frac{1}{1 - \rho} - \frac{1}{2} \right) \left[ \theta^h + \theta^h (\bar{v}_2)^{\rho - 1} \right]$$

and

$$U^f_C = c_1 x_0 \left( \frac{\phi_2}{\theta_2 (\bar{v}_2) - 1} \right)^{1 - \rho} \left( \frac{1}{1 - \rho} - \frac{1}{2} \right) \left[ \theta^f + \theta^f (\bar{v}_2)^{\rho - 1} \right].$$

The Pareto Frontier is derived by solving the problem,

$$\begin{align*}
\max \quad & W = \alpha U^h + (1 - \alpha) U^f \\
\text{s.t.} \quad & 1 \geq a \theta_1 x_1 \bar{c}_1^{\rho - 1} \quad \text{and} \quad 1 \geq a \theta_1 y_1 \bar{c}_1^{\rho - 1}.
\end{align*}$$

Set

$$L = \sum_{k=x,y} \theta_{ax} \left( c_k^{1 - \rho} - a \theta_1 x_1 \bar{c}_1^{\rho - 1} \right) + \lambda_x \left( 1 - a \theta_1 x_1 \bar{c}_1^{\rho - 1} \right) + \lambda_y \left( 1 - a \theta_1 y_1 \bar{c}_1^{\rho - 1} \right),$$

where

$$\theta_{ax} \equiv \alpha \theta^h + (1 - \alpha) \theta^f, \quad \theta_{ay} \equiv \alpha \theta^h + (1 - \alpha) \theta^f.$$

The first-order conditions are

$$\begin{align*}
\frac{\partial L}{\partial M_w} &= \theta_{ax} \left[ c_k^{1 - \rho} - a \theta_1 x_1 \bar{c}_1^{\rho - 1} \right] + \theta_{ay} \left[ c_k^{1 - \rho} - a \theta_1 y_1 \bar{c}_1^{\rho - 1} \right] \\
&\quad - \lambda_x a \theta_1 x_1 \bar{c}_1^{\rho - 1} (1 + \rho) - \lambda_y a \theta_1 y_1 \bar{c}_1^{\rho - 1} (1 + \rho) \\
&= 0 \quad (15)
\end{align*}$$

and

$$\begin{align*}
\frac{\partial L}{\partial M_R} &= -\theta_{ax} a \theta_1 x_1 \bar{c}_1^{\rho - 1} (1 - \gamma) + \theta_{ax} a \theta_1 y_1 \bar{c}_1^{\rho - 1} \gamma \\
&\quad - \lambda_x a \theta_1 x_1 \bar{c}_1^{\rho - 1} (1 - \gamma) + \lambda_y a \theta_1 y_1 \bar{c}_1^{\rho - 1} \gamma \\
&= 0. \quad (16)
\end{align*}$$
Case 1. $\lambda_x > 0$ and $\lambda_y > 0$. We can write

\[ 1 = a\theta_x^{(2\Phi/1-\Phi)}x^2c_x^{\rho-1} \] and

\[ 1 = a\theta_y^{(2\Phi/1-\Phi)}y^2c_y^{\rho-1}. \]

Thus, we reach the CE allocation and (16) implies

\[ \theta_{ax}c_x^{1-\rho}(1-\gamma) + \lambda_x2(1-\gamma) = \theta_{ay}c_y^{1-\rho}(\Upsilon\Theta)^{\rho-1} + \lambda_y2\gamma \]

for any $\alpha$. Thus, we get

\[ \frac{\theta_{ax}}{\theta_{ay}} = \Upsilon^\rho \Theta^\rho. \]

Case 2. $\lambda_x = 0$ and $\lambda_y = 0$. We can write

\[ 1 > a\theta_x^{(2\Phi/1-\Phi)}x^2c_x^{\rho-1} \] and

\[ 1 > a\theta_y^{(2\Phi/1-\Phi)}y^2c_y^{\rho-1}. \]

Since both sectors have monopoly power, this case cannot be consistent with Pareto efficiency.

Case 3. $\lambda_x > 0$ and $\lambda_y = 0$. We can write

\[ 1 = a\theta_x^{(2\Phi/1-\Phi)}x^2c_x^{\rho-1} \] and

\[ 1 > a\theta_y^{(2\Phi/1-\Phi)}y^2c_y^{\rho-1}. \]

Since (15) implies

\[ \theta_{ay}(\Upsilon\Theta)^{\rho-1} \left[ 1 - a\theta_y^{2\Phi}(y^2c_y^{\rho-1}) \right] = \lambda_x(1+\rho)c_x^{\rho-1} \]

and (16) implies

\[ -\theta_{ax}(1-\gamma) + \theta_{ax}a\theta_y^{2\Phi}y^2c_y^{\rho-1}(\Upsilon\Theta)^{\rho-1}\gamma = \lambda_x2(1-\gamma)c_x^{\rho-1}, \]

we get

\[ a\theta_y^{2\Phi}y^2c_y^{\rho-1} = 1 \]

and

\[ a\theta_y^{2\Phi}y^2c_y^{\rho-1} = \frac{1 + (\Upsilon\Theta)^{1-\rho} \frac{\theta_{ax}}{\theta_{ay}} \left( \frac{1+\rho}{2} \right)}{1 + \frac{1+\rho}{2}\Upsilon} \equiv \Lambda < 1. \]

Note that $\Lambda < 1$ if $(\theta_{ax}/\theta_{ay}) < \Upsilon^\rho \Theta^\rho$. Using the results in Table 1, we get

\[ \frac{a\theta_x^{2\Phi}x^2c_x^{\rho-1}}{a\theta_y^{2\Phi}y^2c_y^{\rho-1}} = \left( \frac{M_R}{M_{R0}} \right)^2 = \frac{1}{\Lambda}, \]
and
$$
\left( \frac{M^\Delta_w}{M^\alpha_{w0}} \right)^{\frac{1}{1+\rho}} = \Lambda^{1-\gamma} \frac{\phi \theta_i}{\gamma(\phi - 1)} \left[ \theta_x + \theta_y(\gamma \Theta)^{-1} \right]^{-1}.
$$

Note that since the $x$ sector is binding, the optimal monetary policy will be designated by way of setting $\alpha$, so that the monetary innovation of Home is more expansionary relative to that of Foreign; that is, $(M_R/M_{R0}) > 1$.

Define $U^j_{\alpha k}$ to be the national welfare for country $j$ when the participation constraint of type-$k$ agents is binding in equilibrium. We get
$$
U^h_{\alpha x} = c_0^{1-\rho} \left( \frac{M^\Delta_w}{M^\alpha_{w0}} \right)^{\frac{1}{1+\rho}} \left[ \theta_x^h \left( \frac{1}{1 - \rho} - \frac{1}{2} \right) + \theta_y^h (\gamma \Theta)^{\rho - 1} \right]
$$
and
$$
U^f_{\alpha x} = c_0^{1-\rho} \left( \frac{M^\Delta_w}{M^\alpha_{w0}} \right)^{\frac{1}{1+\rho}} \left[ \theta_x^f \left( \frac{1}{1 - \rho} - \frac{1}{2} \right) + \theta_y^f (\gamma \Theta)^{\rho - 1} \right].
$$

Case 4. $\lambda_x = 0$ and $\lambda_y > 0$. 1 > $a \theta_x^{(2\phi/1-\phi)} x^2 c_x^{\rho - 1}$ and $1 = a \theta_y^{(2\phi/1-\phi)} y^2 c_y^{\rho - 1}$.

Similarly, we get
$$
a \theta_y^{2\phi} x^2 c_x^{\rho - 1} = 1,
$$
$$
a \theta_x^{2\phi} y^2 c_y^{\rho - 1} = \frac{1 + (\gamma \Theta)^{\rho - 1} \theta_x \phi (1+\rho)}{1 + \frac{1+\rho}{2} \gamma^{-1}} \equiv \Omega < 1,
$$
and $\Omega < 1$ iff $(\theta_{ax}/\theta_{ay}) > \gamma \rho \Theta^{\rho - 1}$. Thus,
$$
\frac{a \theta_x^{2\phi} x^2 c_x^{\rho - 1}}{a \theta_y^{2\phi} y^2 c_y^{\rho - 1}} = \left( \frac{M_R}{M_{R0}} \right)^2 = \Omega
$$
and
$$
\left( \frac{M^\Delta_w}{M^\alpha_{w0}} \right)^{\frac{1}{1+\rho}} = \Omega^{\gamma} \frac{\phi \theta_i}{\gamma(\phi - 1)} \left[ \theta_x + \theta_y(\gamma \Theta)^{-1} \right]^{-1}.
$$
Therefore,

\[ U^h_{\alpha y} = c_{x0}^{1-\rho} \left( \frac{M_w^\Omega}{M_w^{00}} \right)^{\frac{1-\rho}{\rho}} \left[ \theta^h_x \left( \frac{1}{1-\rho} - \frac{\Omega}{2} \right) + \theta^h_y (\Upsilon \Theta)^{\rho-1} \left( \frac{1}{1-\rho} - \frac{1}{2} \right) \right], \]

and

\[ U^f_{\alpha y} = c_{x0}^{1-\rho} \left( \frac{M_w^\Omega}{M_w^{00}} \right)^{\frac{1-\rho}{\rho}} \left[ \theta^f_x \left( \frac{1}{1-\rho} - \frac{\Omega}{2} \right) + \theta^f_y (\Upsilon \Theta)^{\rho-1} \left( \frac{1}{1-\rho} - \frac{1}{2} \right) \right]. \]

Note that since now the \( y \) sector is binding, the optimal monetary policy will be designated by way of setting \( \alpha \), so that the monetary innovation of Home is more contractionary relative to that of Foreign, that is, \( (M^R/M^R_0) < 1 \).

Finally, to derive the optimal weight \( \alpha \), the delegate chooses \( \alpha \) to maximize

\[ W_{\alpha k} = \alpha U^h_{\alpha k} + (1 - \alpha) U^f_{\alpha k}, \quad \text{for} \quad k = x, y. \]

The first-order conditions are

\[ U^h_{\alpha x} - U^f_{\alpha x} + \frac{\partial \Lambda}{\partial \alpha} 1 \frac{(1-\gamma)(1-\rho)}{1+\rho} \left[ \alpha U^h_{\alpha x} + (1-\alpha)U^f_{\alpha x} \right] \]

\[ = \frac{1}{2} \frac{\partial \Lambda}{\partial \alpha} c_{x0}^{1-\rho} \left( \frac{M_w^\Lambda}{M_w^{00}} \right)^{\frac{1-\rho}{\rho}} (\Upsilon \Theta)^{\rho-1} \left[ \alpha \theta^h_x + (1-\alpha) \theta^f_x \right], \]

and

\[ U^h_{\alpha y} - U^f_{\alpha y} + \frac{\partial \Omega}{\partial \alpha} \frac{\gamma (1-\rho)}{1+\rho} \left[ \alpha U^h_{\alpha y} (1-\alpha)U^f_{\alpha y} \right] \]

\[ = \frac{1}{2} \frac{\partial \Omega}{\partial \alpha} c_{x0}^{1-\rho} \left( \frac{M_w^\Omega}{M_w^{00}} \right)^{\frac{1-\rho}{\rho}} \left[ \alpha \theta^h_y + (1-\alpha) \theta^f_y \right]. \]

for \( k = x \) and \( k = y \), respectively, where

\[ \frac{\partial \Lambda}{\partial \alpha} = \frac{(\Upsilon \Theta)^{1-\rho} \left( \frac{1+\rho}{2} \right) (\theta^h_x \theta^f_y - \theta^f_x \theta^h_y)}{[1 + (\frac{1+\rho}{2}) \Upsilon] \left[ \alpha \theta^h_x + (1-\alpha) \theta^f_x \right]^2} \]
and

\[
\frac{\partial \Omega}{\partial \alpha} = \frac{(\gamma \Theta)^{\rho-1} \left( \frac{1+\rho}{2} \right) \left( \theta_x \theta_y - \theta_x \vartheta \right)}{\left[ 1 + \left( \frac{1+\rho}{2} \right) \gamma^{-1} \right] \left[ \alpha \theta_y + (1 - \alpha) \theta_x \right]^2}.
\]

References


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國際化生產和貨幣政策協調

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本文建立一個兩國的一般均衡模型來探討在國民化生產 — 廠商可以同時雇用國內外勞動力以製造中間財 — 之下，國際間貨幣政策的協調，對於兩國福利的影響。本文證明各國貨幣當局有誘因偏離原先協調的政策，因此我們進一步分析成立一個共同貨幣當局進行政策協調的效果。我們發現，當國民化生產的程度愈高，共同的貨幣當局所能產生的福利所得會逐漸縮小。因此，中間財的國際貿易降低成立共同貨幣當局的重要性。

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